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Initiating a Mexican wave: An instantaneous collective decision with both short- and long-range interactions

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Abstract

An interesting example for collective decision-making is the so-called Mexican wave during which the spectators in a stadium leap to their feet with their arms up and then sit down again following those to their left (right) with a small delay. Here we use a simple, but realistic model to explain how the combination of the local and global interactions of the spectators produces a breaking of the symmetry resulting in the replacement of the symmetric solution—containing two propagating waves—by a single wave moving in one of the two possible directions. Our model is based on and compared to the extensive observations of volunteers filling out the related questionnaire we have posted on the Internet. We find that, as a function of the parameter controlling the strength of the global interactions, the transition to the single-wave solution has features reminiscent of discontinuous transitions. After the spontaneous symmetry breaking the two directions of propagation are still statistically equivalent. We also investigate how this remaining symmetry is broken in real stadia by a small asymmetrical term in the perception of spectators.

Keywords: Dynamics of social systems; Instantaneous collective decision; Excitable media

1. Introduction

In recent years, the rapid development of observational methods and computational power has made it possible to accumulate data on the *collective motion* of a large number of living organisms [1] and to investigate the few observed universal patterns of motion also by computational tools and *statistical physics-based models* [2–5]. Similar to collective motion, the number of participants in *collective opinion formation and decisions* is often very large; a key factor is interaction (influence and imitation) between the participants, which strongly reduces the number of possible global patterns (see, e.g., Ref. [6]) suggesting that the number of relevant parameters is small. Statistical physics-based models have been successfully applied in the analysis of collective opinion formation and decisions as well. Unanimous and undecided election results and cooperation

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phenomena have been described by models containing particles with a small number of allowed states and simple rules of interaction plus external fields [7–10]. Surprisingly, within the same modelling framework one can explain the shape of the transition measured for the cumulated binary decisions of millions of humans in several further collective decision processes of high public interest, such as birth rate and cell phone purchases [11]. In addition, by allowing the particles to move, one can model, e.g., the spatial separation of opinions [12].

The Mexican wave (also called La Ola), is produced by spectators in a stadium, and it is a well-known example of an instantaneous collective decision. Since its direction of motion is spontaneously selected after a rapid collective decision based on information of limited complexity, it can serve as a paradigm for similar processes. Below, we first give an overview of general observations and data collected on Mexican waves in our online survey. This is followed by a detailed description of the simulation model we have used, including its local and global versions, and later the incorporation of the observed additional left–right asymmetry. Our results on the spontaneous symmetry-breaking transition and the additional left–right asymmetry are in Sections 4 and 5. Additional calculations and the abbreviated list of data we have used (from videos and our online survey) are provided in the Appendices. Note that the full data set is available in the preprint version of this paper from the ArXiv.org server. The same complete data set together with our simulation and evaluation programs can be downloaded from the supplementary website of this paper at http://angel.elte.hu/localglobal.

2. Observations and data on Mexican waves

2.1. General observations

The Mexican wave is launched by a small group of people, each of them standing up within a short time interval, raising their hands high above their heads and then sitting down. As this motion is repeated consecutively by groups of close neighbors, within a few seconds a stable, linear wave with constant amplitude, width and speed develops. Among the reports known to us (see Appendix C) the symmetric solution, i.e., two waves started by the same source and moving in opposite directions, occurs rarely and only by the coordinated action of an experienced group. In a stadium one source can usually trigger only one wave moving either left or right, and the direction becomes clear short after the initiation. The mechanism of this rapid self-organizing process can serve as a paradigm for situations involving limited interaction and the selection of one option out of a small number of possible choices, e.g., route choice behavior in vehicle traffic [13] or the selection of exits during pedestrian escape panic [5].

While the Mexican wave rolls, spectators try to predict when the (nearest) wave will arrive at their seats and leap to their feet at that moment. During stationary propagation around the stadium, both the wave and its velocity are well-defined and spectators can easily synchronize themselves to the wave's arrival time. During the short time interval of the initiation, however, the wave's direction is not yet known. Those who can see that the region of active persons is moving towards them will be more likely to participate than those who find that this region is moving away from them. In other words, a person is activated by the combination of two effects: (i) many of the neighbors are already active and (ii) the nearest active region (wave) is approaching. The first is a short-range effect, while the second is a long-range effect.

2.2. Data on Mexican waves from videos and our online survey

We have evaluated 15 recorded waves from videos and used an online survey to collect additional information about Mexican waves (see Appendix C for details). The waves on the videos were all one-directional, 7 of them propagating in the clockwise and 8 in the counter-clockwise direction. From the 75 visitors of our online survey 46 stated that the wave's preferred direction was clockwise (right to left), 18 that it was counterclockwise (left to right), and 11 mentioned no preferred direction.

An optional question was the geographical location of the visitor. Interestingly, the ratio of votes for clockwise vs. counterclockwise was 32:12 from North America and 5:0 from Europe, while the same ratio was 2:5 from Australia. In addition, some visitors explained in detail that the direction of the wave strongly depends on the relative position of initiators compared to each other and the position of obstacles close to the triggering group (see, e.g., the answers of visitors No. 29 and 73 to the question "Does the wave have a preferred direction?" in Appendix C). The interaction between the spectators in the stadium was claimed to be local, global or both by 15, 37 and 17 people, respectively. From these answers we concluded that (i) usually the wave's motion is influenced by both short-range (watching one's neighbors) and long-range interactions (watching the wave as a whole), and (ii) in most cases an additional left–right asymmetry is also present in the system.

3. Modelling the Mexican wave

As in Ref. [14] we apply an *excitable medium model* to describe the Mexican wave with each particle representing one spectator in the stadium. However, while in Ref. [14] we concentrated on the propagation of the wave, in the present model we intend to capture the main features of the initiation period. Our description is inspired by the Greenberg–Hastings (GH) model of an excitable medium [15]. In the GH model in the beginning of the simulation each particle is in the excitable (also called resting or activable) state. If at time t there is a sufficient number of active particles among the ith particle's neighbors, then the ith particle becomes active (excited) at the next time step, at time $t+\Delta t$. After it has been activated, the GH particle deterministically steps through the n_a active states, and then the n_r refracter states before it returns to its original, excitable state. A particle can be activated only if it is in the excitable state, and only active particles influence other particles.

3.1. Simulation details

For the simulations, the stadium was folded out to a rectangular lattice with $L_x \times L_y$ lattice sites (seats). Rows of seats became parallel to the x-axis, boundaries were periodic in the x direction and non-periodic in the y direction. The positive x direction in this coordinate system corresponds to the clockwise direction in the stadium, while the y-axis is pointing out from the stadium. Stadia for major sport events usually hold 20,000–80,000 seats with 60–100 rows; in the simulations we have used $L_x = 400$ and $L_y = 80$ (corresponding to 32,000 spectators) as a representative size.

In the simulations, the excitable state corresponds to a person sitting and ready to take part in the wave. Active states correspond to raising hands when standing up. Refracter states represent sitting down plus the time interval when the person is already sitting, but not yet activable again. In real situations, in addition to the active and refracter states, there is a short, but finite delay between the time when a person decides to move and the time when he/she actually starts to move. This delay is due to one's reaction time, and we model it by inserting n_d "delay" states between the excitable state and the active states. In summary, in our model after activation a particle is first "waiting" for n_d time steps, it is active during the next n_a time steps, then refracter for n_r time steps, and then it returns to the excitable state.

In one simulation update (one time step) the ith particle is activated with probability $0<p<1$, i.e., moved from state 0 (excitable state) to state 1 (delay state), if (a) it is currently in the excitable state and (b) the total activation effect, W_i, on this particle exceeds the activation threshold, C. The total activation effect, W_i, acting on the ith particle is a combination of local (short-range) and global (long-range) interactions:

$$W_i = G_i \sum_{\substack{j \neq i \\ j \text{ active}}} w_{j \to i}, \tag{1}$$

where G_i is the global interaction strength for the ith particle and the sum contains the local effect, $w_{j\to i}$, of each nearby active particle, j, on the ith particle.

In the simulations for each particle along the $y = L_y/2$ line, the time of the particle's first activation, was saved as a function of the particle's horizontal coordinate, x. After triggering a wave the survival time, t_S, of the wave was defined as the time below which the first activation times showed an increasing function when moving away from the initiating spot both left and right.

3.2. Local version of the model

If we ignore the global interactions in Eq. (1), i.e., we set $G_i = 1$ for each i, then

$$W_i = \sum_{\substack{j \neq i \\ j \text{ active}}} w_{j \to i}. \quad (2)$$

This is the *local version* of the model. A simple form for an isotropic, exponentially decaying *local interaction* with characteristic length R is

$$w_{j \to i} = K_i^{-1} \, \mathrm{e}^{-|\vec{r}_{ij}| \, / \, R}, \quad (3)$$

where $K_i = \sum_m \mathrm{e}^{-|\vec{r}_{im}| \, / \, R}$ is a normalizing constant, and for any given particle, i, the summation goes for all m ($m \neq i$) particles.

In a deterministic case when spectators are identical, the excitable ith particle is activated, if the sum, W_i, of the *local* weights exceeds the activation threshold, C. On the other hand, the activation of spectators in a stadium—just like most processes involving living systems—is *not entirely deterministic*. In the present model this noisy component is taken into account by using a stochastic activation rule for each person: the activation threshold, C, is same for each particle, but the activation of a particle is not deterministic. If for the ith particle the total activation effect, W_i, is above the activation threshold, C, then this particle is activated in the current time step with probability p ($0 < p < 1$). This gives a different response times for each particle.

3.3. Global version of the model

Started with a small group of active particles, the above *isotropic local* version of the model produces—after a transient circular wave phase—two symmetric waves propagating in opposite directions away from the triggering center. However, all video recordings available to us show and an overwhelming majority of our online visitors report that already short after the triggering event only one wave is present. Therefore, an intriguing question is how one of the two waves is suppressed and the other is selected so rapidly. The key effect in selecting the wave's direction in the model so quickly is the *long-range interaction*: if the active region (perturbation, wave) is moving towards (away from) a particle, then this will make the activation of that particle more (less) likely.

To take long-range interactions into account, we computed the average x distance of active particles from the ith particle, $x_i^{(a)}$, using an exponentially decaying weight factor

$$x_i^{(a)} = \frac{\sum^{(a)} \Delta x_{ij} \, \mathrm{e}^{-\Delta x_{ij}/X}}{\sum^{(a)} \mathrm{e}^{-\Delta x_{ij}/X}}. \quad (4)$$

Here the horizontal distance between the ith and jth particles, Δx_{ij}, is the shorter of the two possible distances allowed by the periodic boundary. The characteristic length of the long-range interaction is X ($X \gg R$), and the summation goes for active j ($j \neq i$) particles. Denoting by $v_i^{(a)}$ the time derivative of $x_i^{(a)}$ and by S the sensitivity of spectators to this velocity, the long-range interaction term is

$$G_i(v_i^{(a)}) = \begin{cases} 1, & \text{if } v_i^{(a)} < 0, \\ \mathrm{e}^{-S v_i^{(a)}}, & \text{if } v_i^{(a)} \geqslant 0. \end{cases} \quad (5)$$

Note that $v_i^{(a)}$—the velocity of the active region as seen from the ith person—is positive, if the active region is moving away from this person, and it is negative, if the active region is approaching the ith person. In the $S \to 0$ limit, G_i is a step function and the decision about the direction of the wave is very sharp. In this case one of the two directions is selected quickly and the wave in the other direction is suppressed, because particles on that side "measure" a positive $v_i^{(a)}$, and, consequently, $G_i = 0$ and $W_i = 0$. In the $S \to \infty$ limit the global interaction term will be a constant, $G_i = 1$, which gives the local version of the model (Eqs. (2) and (3)). In the

$0 < S < \infty$ case an approaching wave, i.e., $v_i^{(a)} < 0$, will make the ith person more likely to participate, while a departing wave less likely.

3.4. Default parameter values

Simulation updates were parallel and synchronized, and the time step was constant, 0.1 s. We started each simulation by triggering a small group of particles: for each particle inside a circle with radius $\rho = 3$ and centered at $(L_x/2, L_y/2)$, we selected a (discrete) time point randomly from the interval (0 s,1 s), and moved the particle at this time point from state 0 (excitable state) to state 1 (delay state). The time derivative, $v_i^{(a)}$, was computed with Euler's formula and a time step of $\Delta t = 0.5\,\mathrm{s}$. If at time t for the ith particle either $x_i^{(a)}(t)$ or $x_i^{(a)}(t - \Delta t)$ was not available, then the long-range interaction coefficient, $G_i(v_i^{(a)})$, was replaced by 1. Default parameter values were $R = 3$, $X = 100$, $C = 0.23$, $p = 0.9$, $n_\mathrm{d} = 1$, $n_\mathrm{a} = 10$, $n_\mathrm{r} = 20$. Distances were measured in seats, time was measured in seconds. To compute the local weights, we applied a cutoff and restricted the summation to $|\vec{r}_{ij}| \leqslant 3R$.

4. Spontaneous symmetry breaking

In our model the relative weight of global interactions is given by S. In the $S \to 0$ limit one obtains the isotropic local version of the model, where the stable solution contains two oppositely moving waves. Raising S causes a spontaneous symmetry breaking (global interactions are "turned on"), and the symmetrical solution becomes unstable. If S is high, then soon after the initiation, one of the two directions is selected and propagation in the other direction is stopped. Thus, S changes the symmetry properties of the stable solution and can be used as a *control parameter*. Fig. 1 shows how at different values of S either two oppositely moving waves develop from one initiating source or one of the two directions is selected.

Since the control parameter, S, tunes the stability of the symmetric solution, we have selected an *order parameter* measuring this stability. For a wave starting off asymmetrically at the triggering spot the survival

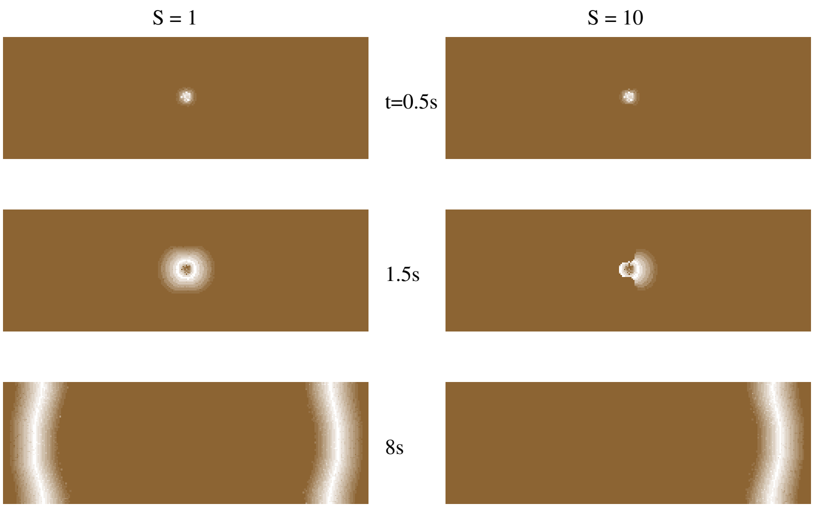

Fig. 1. Spontaneous symmetry breaking in the Mexican wave simulations: each spectator is represented as one particle in a rectangular lattice. Shown are parts of the simulation area at 0.5, 1.5 and 8 s after the triggering event (see Section 3.4 for the default parameter values). Excitable particles are colored dark. The increasing color brightness of active particles represents the different stages while standing up, the decreasing color brightness of the first 10 refracter states represents the stages of sitting down, while the dark color of the remaining 10 refracter states indicates that the person is already sitting, though not yet activable again. **Left column.** If the control parameter, S (which is also the relative weight of global interactions), is low, then the stable solution contains two waves moving in opposite directions. **Right column.** At higher values of the control parameter the asymmetric solution—one wave moving either left or right—becomes stable.

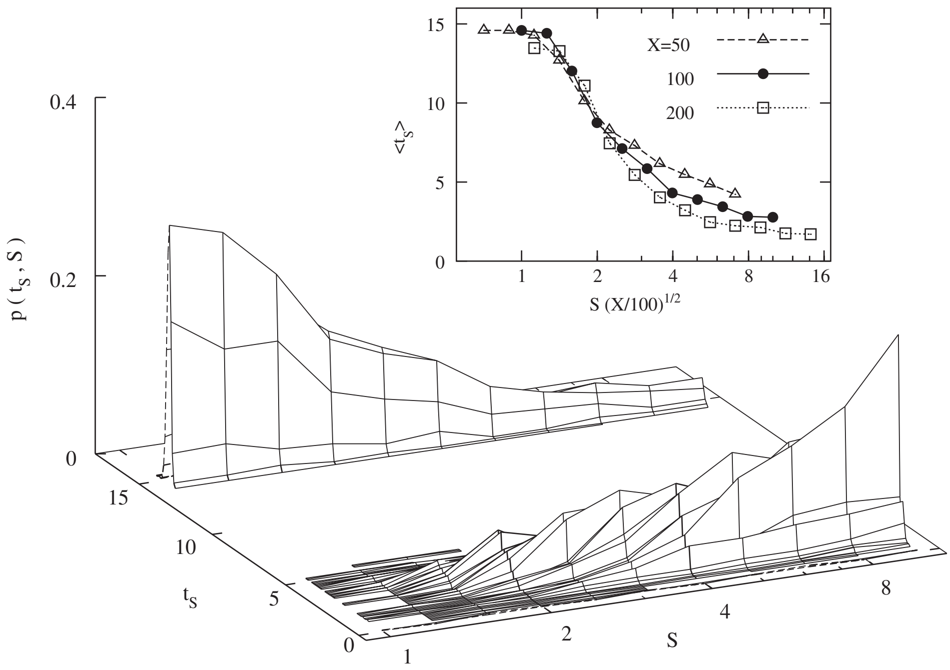

Fig. 2. **Main panel.** Transition between the symmetric (two waves moving in opposite directions) and asymmetric solutions (one wave moving either left or right) in the Mexican wave model. The control parameter, *S*, which is the parameter of global asymmetry in the model, is analogous to the inverse temperature, β, of temperature-controlled transitions. For each *S* the distribution of t_S ("survival time") is displayed: t_S is the time until which the two oppositely moving waves are both present in the system. **Inset.** The average survival time, $\langle t_S \rangle$, of the symmetrical solution for different values of the global interaction length, *X*. The transition point, S_C, scales as $X^{-1/2}$, and the transition itself becomes sharper with increasing *X*. Data points show averages over 1000 simulations for each value of *S*.

time is $t_S \approx 0$. On the other hand, for an infinitely stable symmetrical solution the survival time is a constant finite value corresponding to the time needed for the two waves to meet at the opposite end of the stadium. The main panel of Fig. 2 shows the distribution of t_S values as a function of the control parameter, *S*. We found that for a range of *S* values the distribution of t_S has two distinct peaks, which is analogous to the distribution of the order parameter in the vicinity of the transition point in systems undergoing a discontinuous transition. The two phases are the symmetric solution with two waves (high t_S) and the asymmetric solution is a wave moving either left or right (low t_S). Since the size of the initiating group is always finite (in the range of a few dozen particles), the phenomenon is inherently mesoscopic and the transition is not very sharp.

The inset of Fig. 2 shows the average of the order parameter, $\langle t_S \rangle$, as a function of *S* at different values of the long-range interaction length, *X*. The transition point scales as $X^{-1/2}$, and the transition becomes sharper with increasing *X*. This scaling is caused by (i) the uncorrelated random activation of particles—both in space and time—in the triggering group and (ii) the locally linear shape of the global interaction term as a function of the horizontal coordinate (see Eq. (5)). A detailed derivation of this scaling is provided in Appendix B. During the triggering the expected speed of the active region, $v_i^{(a)}$,—as seen from a nearby excitable particle, *i*, scales as $X^{1/2}$. Since the global interaction term is a function of $Sv_i^{(a)}$, this gives $S_C \sim X^{-1/2}$ (see Appendix B for details).

5. Left–right symmetry breaking

The above description assumes that during the triggering of the wave, the participating spectators stand up in an uncorrelated fashion. In real situations, however, additional, very fast interactions are present, and the decision about the direction of the wave is usually faster than predicted here. It is well known that people inside the triggering spot influence each other both before they start moving and also during the very short time interval of the triggering. From the responses to our online survey (see Appendix C) we found that one important additional effect influencing the wave's direction and stability is the local geometry, e.g., the presence of obstacles around the triggering spot. Also, people tend to react asymmetrically to disturbances occurring on their left and right sides; this is caused by our physiological asymmetry and our expectations. If by a combination of these effects people react stronger to stimuli on, e.g., their left, than to those on their right, then a wave will most likely propagate from the left to the right, which is the counterclockwise direction in the stadium when watched from above.

To model the inherent local asymmetry during the triggering, we extended the global version of the model and changed Eq. (3). In the local coordinate system of the ith particle, let φ denote the angle of $\vec{r}_{ij}$ pointing from the ith particle to the jth particle. If $\vec{r}_{ij}$ points to the left (the clockwise direction when watched from above), then $\varphi = 0$, and for $\vec{r}_{ij}$ pointing radially out from the stadium, $\varphi = \pi/2$. We used the following direction-dependent local interaction term (compare to Eq. (3)):

$$w_{i\to j} = K_i^{-1}\, e^{-|\vec{r}_{ij}|/R}\,[(1-\delta) + \delta\cos(\pi - \varphi)]. \tag{6}$$

Similar to Eq. (3), K_i is a normalizing constant chosen such that for the ith particle the sum of $w_{i\to j}$ $(j \neq i)$ values is 1.

In the model, "switching on" the left–right asymmetry of local interactions has two simultaneous effects. (i) The balance between left and right moving waves will be broken (see the main panel of Fig. 3): by increasing δ the probability of the right-moving wave, P_R, will be dominant over the probability of the left-moving wave, P_L. (ii) The spontaneous symmetry breaking transition will be shifted (inset of Fig. 3). The inset of Fig. 1 shows that for all values of X, the transition point of the spontaneous symmetry breaking is at approximately $S_C(X) = 2(X/100)^{-1/2}$. This is the value of S, where the average survival time of the symmetric solution (two waves), $\langle t_S\rangle$ reaches half of its maximum value. However, with the same S values and $\delta > 0$, the survival time of

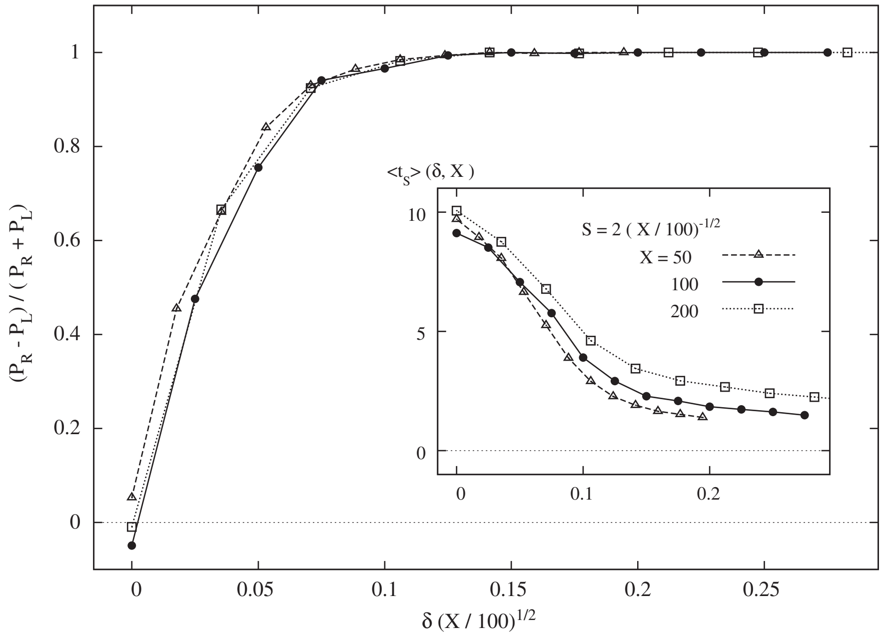

Fig. 3. Left–right symmetry breaking in the Mexican wave simulations caused by the local asymmetry during the triggering of the wave. The $\delta = 0$ case corresponds to the isotropic local version of the model (see Eqs. (2) and (3)). The sensitivity of particles to the speed of the active region is $S = 2\,(X/100)^{-1/2}$, i.e., the system is at the transition point (see the inset of Fig. 2). **Main panel.** The difference between the probabilities of left (P_L) and right (P_R) moving waves as a function of the local left–right asymmetry parameter, δ. **Inset.** The average survival time, $\langle t_S\rangle$, for different values of the long-range interaction length, X. Data points show averages over 1000 simulations for each value of δ.

the symmetric solution becomes significantly smaller, i.e., the symmetric solution is destabilized by the left–right asymmetry term.

Observe that in both cases the transition curves collapse when plotted as a function of $\delta X^{1/2}$, i.e., the transition point, δ_C, scales with the global interaction length as $X^{-1/2}$. This is the same scaling as the one that we have seen earlier for S_C in the spontaneous symmetry breaking, and it has the same two reasons (see Appendix B for a detailed derivation): the random activation of particles in the triggering spot is uncorrelated and the left–right asymmetry term is a locally linear function.

6. Conclusions

We have presented a simple realistic model of an instantaneous collective human decision process, where the interplay of local and global interactions leads to a spontaneous symmetry breaking. The decision we have modelled concerns the direction of propagation of the Mexican wave (La Ola) in a stadium after a small group of people stands up to initiate the wave. Although the situation and the model we have considered is relatively simple, they give an insight into the mechanisms by which quick decisions are made by groups of people. Understanding such phenomena is important in various contexts including the spreading of excitement in panicking crowds or during collective actions of humans at gatherings and during collective financial decisions.

Acknowledgments

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Appendix A. Simulation software

The simulation program was run on Linux computers. Its core performing the numerical operations was written in C + + and the graphical interface in Qt. The simulation program can be run both with and without visualization. The additional scripts and utilities that can start the simulation program at various parameter values and evaluate the results were written in Perl and C. Our programs—including their source codes and a short documentation—can be downloaded from our website at http://angel.elte.hu/localglobal.

Appendix B. Scaling of the transition point in the spontaneous symmetry breaking

To interpret the $S_C \sim X^{-1/2}$ behavior (see the inset of Fig. 2) consider the ith particle, just outside the initial activation spot (a circle with radius ρ). The horizontal (x) distance of this particle from the center of the spot is ℓ. When the simulation is started, the ith particle is affected by the active particles, j, inside the spot. Since $|\Delta x_{ij} - \ell| \leqslant \rho \ll X$, the exponential weight function in Eq. (4) can be approximated with a linear function:

$$x_i^{(\mathrm{a})} = \frac{\sum^{(\mathrm{a})} \Delta x_{ij}\, \mathrm{e}^{-\ell/X}\, \mathrm{e}^{-(\Delta x_{ij}-\ell)/X}}{\sum^{(\mathrm{a})} \mathrm{e}^{-\ell/X}\, \mathrm{e}^{-(\Delta x_{ij}-\ell)/X}} \simeq \frac{\sum^{(\mathrm{a})} \Delta x_{ij}\,[1-(\Delta x_{ij}-\ell)/X]}{\sum^{(\mathrm{a})}[1-(\Delta x_{ij}-\ell)/X]}. \tag{7}$$

The $\sum^{(\mathrm{a})}$ summations run for active particles, j ($j \neq i$), inside the triggering spot. Denoting $(\Delta x_{ij} - \ell)/X$ by η_{ij}, and the average by an overline, one obtain

$$x_i^{(\mathrm{a})} - \ell = X - X\frac{\sum^{(\mathrm{a})}(1-\eta_{ij})^2}{\sum^{(\mathrm{a})}(1-\eta_{ij})} = X - X\frac{1-2\overline{\eta}+\overline{\eta^2}}{1-\overline{\eta}}.$$

Note that from this point on each average containing η has an additional index, i. Since $|\eta_{ij}| \ll 1$, we can drop the $\mathcal{O}(\overline{\eta}^2)$ and $\mathcal{O}(\overline{\eta^2})$ terms:

$$x_i^{(\mathrm{a})} - \ell \cong \overline{\eta}\, X. \tag{8}$$

During the initial triggering, activations occur with uniform spatial distribution inside the spot, and also with uniform distribution in time. Moreover, the expected distribution of η_{ij} values is symmetrical around 0 at any time point. Thus, the process of the summation of η_{ij} values is always a random walk around 0, and the expected sum is proportional to the square root of the linear scale of the summed values. The variable part of η_{ij} is $\Delta x_{ij}/X$. Therefore, the linear scale is proportional to X^{-1}, and the changing part of the above sum scales as $X^{-1/2}$. Similarly, the changing part of the average, $\overline{\eta}$, and the expected rate of change of the average, $(\mathrm{d}/\mathrm{d}t)\overline{\eta}$, are both proportional to $X^{-1/2}$. Differentiating Eq. (8) with respect to time gives $v_i^{(\mathrm{a})}$ on the left and $X\,(\mathrm{d}/\mathrm{d}t)\overline{\eta}$ on the right hand side, and we get $v_i^{(\mathrm{a})} \sim X^{1/2}$. The global interaction term is a function of $Sv_i^{(\mathrm{a})}$, and so the product $S_C\, v_i^{(\mathrm{a})}$ should be constant when X is changed. Therefore, $S_C \sim X^{-1/2}$.

The above approximation for the speed of the active region, $v_i^{(\mathrm{a})}$, is valid only during the triggering. The $v_i^{(\mathrm{a})} \sim X^{1/2}$ proportionality means that (i) the decision process is slightly faster for a larger global interaction length and (ii) with a weak global interaction ($X \to 0$) there is no spontaneous symmetry breaking.

Appendix C. Data on Mexican waves

Evaluation of recorded Mexican waves (videos)

Table 1 lists the 15 recorded Mexican waves we have evaluated: 14 of them were waves rolling among the spectators in stadia and indoor halls built for athletic events and holding up to 50,000 spectators, while 1 of the waves (the file is called sportwav.MPG) was performed on a sports field by a group of approximately 50 children. On all videos only one wave—moving in one of the two possible directions—can be seen, in 7 cases it was a clockwise wave and in the remaining 8 cases it was moving in the counter-clockwise direction.

Answers collected in our online survey (for the complete list of answers, see the preprint version at ArXiv.org)

We have set up an online survey on Mexican waves at http://angel.elte.hu/wave. → select *SURVEY* in the top right corner. The answers to this survey between July 2004 and August 2005 are listed below. The question

Table 1
Direction and duration of 15 recorded Mexican waves evaluated from videos

File name	Duration [s]	Direction	Source
laola01.mpeg	14	Counterclockwise	D.H.
laola02.mpeg	7	Clockwise	D.H.
laola03.mpeg	10	Clockwise	D.H.
laola04.mpeg	11	Clockwise	D.H.
laola05.mpeg	10	Clockwise	D.H.
laola06.mpeg	8	Counterclockwise	D.H.
laola13.mpeg	7	Clockwise	D.H.
laola14.mpeg	14	Clockwise	D.H.
laola15.mpeg	14	Counterclockwise	D.H.
laola16.mpeg	14	Counterclockwise	D.H.
laola17.mpeg	14	Counterclockwise	D.H.
laola21.mpeg	2	Counterclockwise	D.H.
laola22.mpeg	6	Counterclockwise	D.H.
Mvc-528w.mpg	12	Clockwise	web1
sportwav.MPG	5	Counterclockwise	web2

See Section 2.2 of the paper and Appendix C for a detailed analysis. Sources: D.H.: Dirk Helbing, web1: derat.nl, web2: web.ukonline.co.uk/Members/s.livingston.

about the geographical location of the visitor was added in September 2004. A detailed evaluation of the answers is given in Section 2.2 above.

Abbreviated list of answers from the online survey

(1) Jul 14, 2004

* **Have you ever taken part in a Mexican wave?**
YES

* **If yes, did you follow your neighbours or you considered rather the motion of the wave as a whole? Please, explain below.**
As a whole.

* **Does the wave have a preferred direction? (Clockwise or counter-clockwise?)**
yes. clockwise.

* **Have you ever seen two waves initiated by a single source? (i.e., the wave leaving in both directions) If yes, please give details, e.g., when and where.**
do not remember.

* **Have you ever taken part in initiating a wave? What did you do? How many of you were needed? Did you try to influence the direction of the wave from the very beginning?**
no.

...

(29) Mar 14, 2005

* **Have you ever taken part in a Mexican wave?**
YES

* **If yes, did you follow your neighbours or you considered rather the motion of the wave as a whole? Please, explain below.**
the wave as a whole

* **Does the wave have a preferred direction? (Clockwise or counter-clockwise?)**
No its dependent on where the initiator, (i.e., the person lively enough to want to start the wave), sits in relation to his friends. Since it cant be started by one person, not notifying a few people in row beforehand, the initiator will more than likely get the people he knows at the furthest end to where her sitting start it. Therefore it is dependent on the initiators position among his friends.

* **Have you ever seen two waves initiated by a single source? (i.e., the wave leaving in both directions) If yes, please give details, e.g., when and where.**
No but that a great idea

* **Have you ever taken part in initiating a wave? What did you do? How many of you were needed? Did you try to influence the direction of the wave from the very beginning?**
Yes on a trip to Yankee Stadium New York (although I'm Irish). I was there in a big group of international students I got on well with. Proudly it was my idea to start it and yes the direction was pretty much set out since we got a few of us in a row to do it. I remember we did it a few times since the first did not get a reaction. The second time I think there was one random guy who did it and we cheered him. Then since I presume people around us were catching on to what we were doing the third time it went to the end of the stadium. Now I say end of the stadium because Yankee Stadium (a base ball stadium) is a crest shape i.e., its not a complete circle. Regardless I think a lot of people were starting to see the little waves and a lasting wave was started (sadly not by myself but somewhere else). This wave when it reached the side of the stadium the other side kept it going and around and around it went. I still feel I laid the ground work for this lasting wave though.

* **Please, select your geographical location.**
Europe

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(73) Aug 11, 2005

* **Have you ever taken part in a Mexican wave?**
YES

* **If yes, did you follow your neighbours or you considered rather the motion of the wave as a whole? Please, explain below.**
  I think its a function of the two. As I see it, in a normal stadium you are able to see very well the wave approach from afar, but it is more difficult to see where the wave is as it quickly approaches you. I considered the motion of the wave as a whole for timing of when it would approximately get to me and then I used my neighbours as the key signal as to when I should stand.
* **Does the wave have a preferred direction? (Clockwise or counter-clockwise?)**
  I think it depends on the stadium and the sparking group. My last wave experience it went counter-clockwise. Although i could not see where it started, I think the sparking group was in the top level of left field. There is a gap between this area and right field. The wave must go counter-clockwise if it is started there. It would be interesting to include a description of the stadium and the location of the waves starting point in relationship to direction.
* **Have you ever seen two waves initiated by a single source? (i.e., the wave leaving in both directions) If yes, please give details, e.g., when and where.**
  No. I also think it would shorten the number of revolutions due to the law of diminishing returns.
* **Have you ever taken part in initiating a wave? What did you do? How many of you were needed? Did you try to influence the direction of the wave from the very beginning?**
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* **Please, select your geographical location.**
  North America
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