Human waves in stadiums

I. Farkas[a], D. Helbing[b], T. Vicsek[a,*]

[a]*Department of Biological Physics and Biological Physics Research Group of HAS, Eötvös University, Pázmány Peter Setany 1A, Budapest H-1117, Hungary*
[b]*Institute for Economics and Traffic, Dresden University of Technology, Dresden D-01062, Germany*

Abstract

Mexican wave first widely broadcasted during the 1986 World Cup held in Mexico, is a human wave moving along the stands of stadiums as one section of spectators stands up, arms lifting, then sits down as the next section does the same. Here we use variants of models originally developed for the description of excitable media to demonstrate that this collective human behaviour can be quantitatively interpreted by methods of statistical physics. Adequate modelling of reactions to triggering attempts provides a deeper insight into the mechanisms by which a crowd can be stimulated to execute a particular pattern of behaviour and represents a possible tool of control during events involving excited groups of people.

Interactive simulations, video recordings and further images are available at the webpage dedicated to this work: http://angel.elte.hu/wave.

1. Introduction

The application of ideas, methods and results of statistical physics to a wide range of phenomena occurring outside of the realm of the non-living world has gained a great momentum recently. Among many others, examples include the studies of various group activities of people from the physicist's viewpoint. Here, we give an account of an investigation in this direction, involving the interpretation of a wave spontaneously produced by a very large number of people.

---

* Corresponding author.

E-mail address: vicsek@angel.elte.hu (T. Vicsek).

Our motivation originates in the observation that there has been a growing interest in a better, more exact understanding of the mechanisms underlying the main processes in societies. There is a clear need for the kind of firm, reliable results produced by natural sciences in the context of the studies of human behaviour [1]. The revolution in information and transportation technology brings together larger and larger masses of people (either physically or through electronic communication). New kinds of communities are formed, including, among many others, internet chat groups or huge crowds showing up at various performances, transportation terminals or demonstrations. The way people behave in stadiums represents a well defined and relatively simple example for related collective human behaviour.

The term "Mexican wave" was first used during the 1986 World Cup held in Mexico and since then it has become a paradigm for a variety of phenomena in which an initial perturbation propagates in the form of a single "planar" wave. The increasing popularity of this phrase, also known as La Ola, is due to its unique origin; it means a human wave moving along the stands of stadiums as one section of spectators stands up, arms lifting, then sits down as the next section does the same. The corresponding wave at a given instance involves a coordinated behaviour of hundreds, and all together tens of thousands of people as the wave makes its turn around the stadium.

For a physicist, the interesting *specific feature* of this spectacular phenomenon is that it represents perhaps the simplest spontaneous and reproducible behaviour of a huge crowd with a surprisingly high degree of coherence and level of cooperation. In addition, La Ola raises the exciting question of the ways by which a crowd can be stimulated to execute a particular pattern of behaviour.

2. Observations

We have *collected video recordings* of waves in stadia holding over 50,000 people. We found that the Mexican wave can be decomposed into two phenomena: the formation of the wave and the propagation of the wave. The wave is usually formed by the simultaneous standing up of a small number of spectators. The spectators standing up in this group represent a small *perturbation* (excitation), which—depending on their neighbours—may quickly spread out. However, this spreading will occur only, if the audience is in the "appropriate mood" for a wave: not too bored, but not too excited either. In other words: the audience should be sitting, but "excitable". If the perturbation is amplified, then—according to our observations (14 videos of Mexican waves)—during the following short time interval (less than a second) the wave will die out on one side, and survive on the other. The parameters describing the reactions of spectators to nearby excitations, e.g., neighbours jumping to their feet, is described below.

The *spontaneous symmetry breaking* observed during the formation of the wave is probably due not only to the asymmetrical nature of human perception, but also to the expectations of spectators with previous experience on Mexican waves and other forms of collective human behaviour. According to video recordings we have analysed, approximately three out of four Mexican waves move in the clockwise direction, while
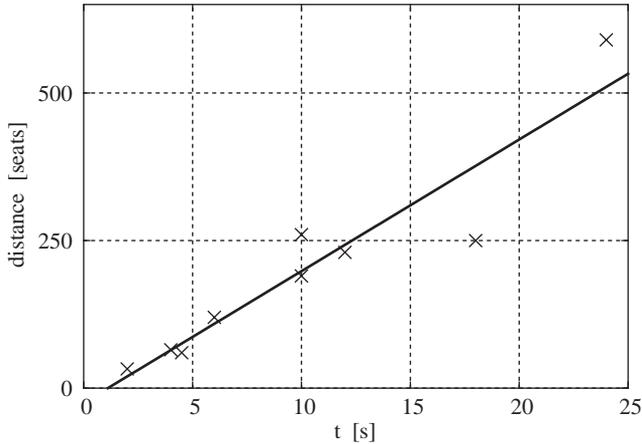
Fig. 1. Speed of the Mexican wave. Each point shows the distance between the final and starting position of a wave on one of the video recordings versus the time needed by the wave to cover that distance. A linear fit, $s = vt + s_0$, to the data points is shown with a solid line. The fit parameters are $v = (22 \pm 3)$ seats/s and $s_0 = (-25 \pm 37)$ seats.

one out of four waves moves in the anti-clockwise direction. Being aware that numerous psychological and physical phenomena influence this symmetry breaking, we decided that for our simulations we will use the simplest model that can reproduce the (i) formation of the wave by triggering and spontaneous symmetry breaking and (ii) the stable propagation of the wave.

To measure the *speed of the waves*, we did the following. For each wave, we noted the wave's starting and final position on the recording and measured the time that the wave needed to cover this distance. In Fig. 1, the distance travelled by each wave is plotted against the time difference between the starting and final position of the wave on the video. Out of the 14 recorded waves available to us nine were of a quality that allowed for the evaluation of their speed. (The remaining five waves were either recorded for a very short time or had irregular shapes.)

As shown in Fig. 1, we have found that the number of seats travelled by a wave is—within experimental error—a linear function of the elapsed time, and the speed is 22 ± 3 seats/s. In addition to measuring the speed of the waves and finding the direction of propagation, we recorded that the typical width of a wave was 6–12 m (equivalent to an average width of approximately 15 seats).

3. Simulation models

The relative simplicity of the phenomenon allowed us to develop microscopical models of the motion of inidividuals, and to reproduce the observed macroscopical quantities with these models. We have found that well-established approaches to excitable media [2–5] such as forest fires or wave propagation in heart tissue can be generalized

Table 1
States of a spectator: inactive, active or refracter

State	Can be activated?	Can activate?
Inactive	Yes	No
Active	No	Yes
Refracter	No	No

to include the behaviour of human crowds, and for this reason we started building our models from the descriptions used in these fields. Interactive simulations, video recordings and further images are available in Refs. [6,7].

3.1. General description of the models

At time t person i can be in one of the following states: inactive, active or refracter. In the inactive (ground) state a person is sitting, and can be activated. An active person is moving upwards (standing up) and is not influenced by others. In the refractory state a person is sitting down or already sitting while passively recovering from the previous activity. In summary, people can be influenced (activated) only in the inactive state, and they influence other people only if they are in the active state (see Table 1).

During one simulation update first the weighted concentration of active people within a fixed radius around each person is compared to that particular person's activation threshold. Next, if a person is inactive, then he/she can be activated using the result of this comparison and the respective model's activation rule. Each person needs a reaction time τ to update the weighted concentration of active people around him/herself.

Weights are proportional to the cosine of the direction angle of the vector pointing from the influenced inactive person to the influencing active person. People exactly to the left from a person have an influence w_0 times ($w_0 < 1$) as strong as those exactly to the right. Weights decay exponentially with distance: the decay length is R, and there is a cutoff at $3R$. The sum of weights is one for each person.

3.2. Detailed description of the models

We are using a rectangular simulation area with 80 rows and 800 "columns" of seats (units, or people). In the x direction (the tangential direction in the stadium) boundary conditions are periodic, and in the y (radial) direction boundary conditions are non-periodic. The simulation is visualised by arranging the spectators evenly on the stadium-shaped tribune.

Each person is facing the inside of the stadium. The coordinate system is always local, and it is fixed to a person such that the vector $(1, 0)$ points towards the left of the person (i.e., parallel to the rows of the stadium in the clockwise direction, if viewed from above), and the vector $(0, 1)$ points towards the back of the person (radially out from the stadium, if viewed from above).

The angle of the vector $\vec{r}_{ij}$, pointing from person i to person j, is φ_{ij}. The angle of the vector $(1,0)$ is $\varphi_{ij} = 0$, and the angle of the vector $(0,1)$ is $\varphi_{ij} = \pi/2$. The interaction decay length is R, and there is a cutoff at $3R$. If person j_{left} is to the left from person i and person j_{right} to the right at the same distance, then the ratio of influence strengths from j_{left} vs. j_{right} is w_0. If one compares all possible directions, the strongest influence comes from the right (the direction $\varphi_{ij} = \pi$), and the weakest influence from the left (the direction $\varphi_{ij} = 0$), and between them the strength changes with the cosine of $\pi - \varphi_{ij}$.

In summary, the weight of person j's influence on person i is

$$w_{ij} = S_i^{-1} \exp(-|\vec{r}_{ij}|/R)[1 + w_0 + (1 - w_0)\cos(\pi - \varphi_{ij})] \,,$$

if $0 < |\vec{r}_{ij}| < 3R$, and $w_{ij} = 0$ otherwise .

The normalizing constant, S_i is defined such that the sum of all influence weights on person i is 1:

$$1 = \sum_j w_{ij} \,.$$

The weighted concentration of active people felt by person i is

$$W_i = \sum_{j \text{ active}} w_{ij} \,.$$

In all three models, the activation of person i depends on whether W_i exceeds the activation threshold of this person, c_i. The reaction time of a person is τ. This is the time difference between two updates of the sum of weights, W_i, felt by the ith person.

3.3. Variants of the model

We have used three variants of the Mexican wave model: (i) version A of the detailed n-state model, (ii) version B of the detailed n-state model and (iii) the minimal three-state model.

The *detailed n-state model* divides the active state into n_a substates and the refractory state into n_r substates. Once activated, a person will deterministically step through the n_a active states, then through the n_r refractory states, and last it will return to the inactive state.

In *version A* of the detailed n-state model a person will be activated under the following conditions. If the weighted concentration of neighbouring active people is above a person's threshold, then the person is activated. Individual activation thresholds are used for each person. The threshold values are randomly and uniformly selected from the interval $(c - \Delta c, c + \Delta c)$. (In the parameter window of the simulation program Δc is written as "delta c".)

In *version B* of the detailed n-state model the activation threshold of person i is c (a constant between 0 and 1). If the weighted concentration of neighbouring active people (W_i) is below c, then the person can be activated spontaneously with probability p_{spont}. If $W_i > c$, then the activation of the person will be induced with probability p_{ind}.

Fig. 2. Simulation model of the Mexican wave. The interactive simulations that were used to produce this image are available in color at http://angel.elte.hu/wave. The simulation is shown 1, 2, 3, 5 and 15 s after the triggering event. The darkest color shows inactive people. The color of active people turns gradually lighter as they stand up, and the color of inactive people turns again darker, as they sit down. The simulation model used for this image was obtained with version A of the detailed *n*-state model. Parameter values were $n_a = n_r = 5$, $c = 0.25$, $\Delta c = 0.05$, $R = 3$, $w_0 = 0.5$ and $\tau = 0.2s$. See the detailed *n*-state model in Section 3.3 below for the explanation.

In the *minimal three-state model*, if the weighted concentration of neighbouring active people around a person is below c, then the person is activated spontaneously with probability p_{01}. If $W_i > c$, then activation will be deterministic (with probability 1). The transition from the active to the refractory state takes place with probability p_{12} at each simulation update, and the transition from the refractory to the inactive state with probability p_{20} (see Fig. 2).

4. Results

As an application, we computed the conditions necessary for the triggering of a Mexican wave in a stadium. The size of the stadium we analyzed was 80 rows of seats with 800 seats in each row. For the simulation, we used a square lattice of size 80×800 instead of the stadium-shaped tribune. The stadium shape was used only for visualization purposes. Each field of the square lattice contained one seat (representing one spectator in the stadium) and the simulation was updated with a parallel update method.

We computed the dependence of the probability of triggering a wave as a function of two different pairs of parameters. (In both cases, we used version A of the detailed *n*-state model.) First, we changed the size of the triggering group (N) and the activation threshold (c) and found that at $N = 30$, $c = 0.25$ a wave could be triggered with almost unit probability [6]. Other parameters were $n_a = n_r = 5$, $\Delta c = 0.05$, $R = 3$, $w_0 = 0.5$ and $\tau = 0.2$. During the second analysis simulations were started with the same parameters as in the first one except for the two variables: the activation threshold (c) and the variations of the activation threshold (Δc). The $N = 30$, $c = 0.25$, $\Delta c = 0.05$ point is shown again as the threshold for wave triggering and a higher variability in the thresholds can maintain waves at lower average thresholds (see Fig. 3). The explanation for this might be quite simple: even when the average activation threshold is low, a small number of people that are easy to activate can strongly increase the chances for the survival of a perturbation in the form of a full-grown wave.
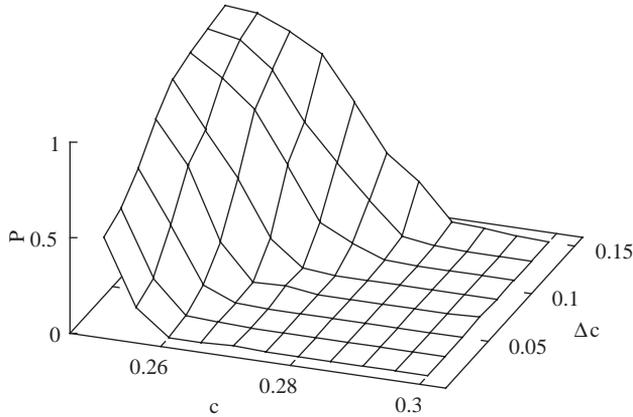
Fig. 3. The ratio of successful triggering events, $P(c, \Delta c)$, where P is the probability that a wave can be induced; c and Δc are the average activation threshold and the differences in activation thresholds among people (for a more detailed definition, see version A of the detailed n-state model in Section 3.3). Other parameters were identical to those of Fig. 2. Our (numerical) definition for the necessary condition of the survival of a wave was the following: 20 s after the triggering event there has to be a column of spectators (a line of spectators sitting behind each other) out of whom at least 50% are either active or refracter (i.e., not passive). Interactive simulations are available at http://angel.elte.hu/wave.

Acknowledgements

References

[1] J.S. Coleman, Foundations of Social Theory, Belcamp Press, Cambridge, MA, 1990.
[2] N. Wiener, A. Rosenblueth, The mathematical formulation of the problem of conduction of impulses in a network of connected excitable elements, specifically in cardiac muscle, Arch. Inst. Cardiol. Mexico 16 (1946) 205–265.
[3] J.M. Greenberg, S.P. Hastings, Spatial patterns for discrete models of diffusion in excitable media, SIAM J. Appl. Math. 34 (1978) 515–523.
[4] J.J. Tyson, Travelling waves in excitable media, in: B.D. Sleeman, R.J. Jarvis (Eds.), Ordinary and Partial Differential Equations II, Longman Scientific & Technical, Harlow, UK, 1989.
[5] G. Bub, A. Shrier, L. Glass, Spiral wave generation in heterogeneous excitable media, Phys. Rev. Lett. 88 (2002) 058 101.
[6] I. Farkas, D. Helbing, T. Vicsek, Mexican waves in an excitable medium, Nature 419 (2002) 131.
[7] The webpage http://angel.elte.hu/wave dedicated to the present work offers further data and interactive simulations.