Origins of hierarchical cooperation

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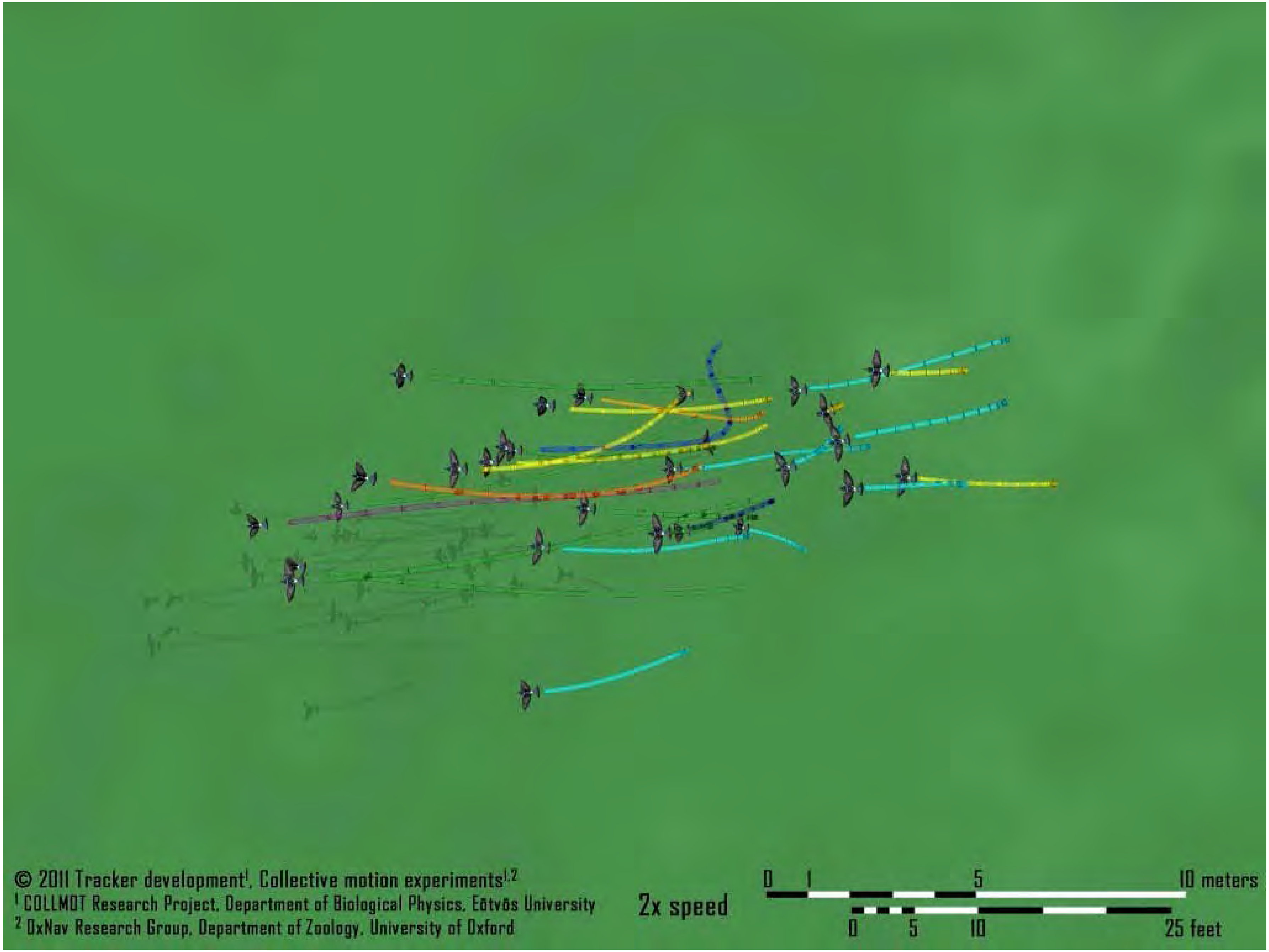
Motivation I

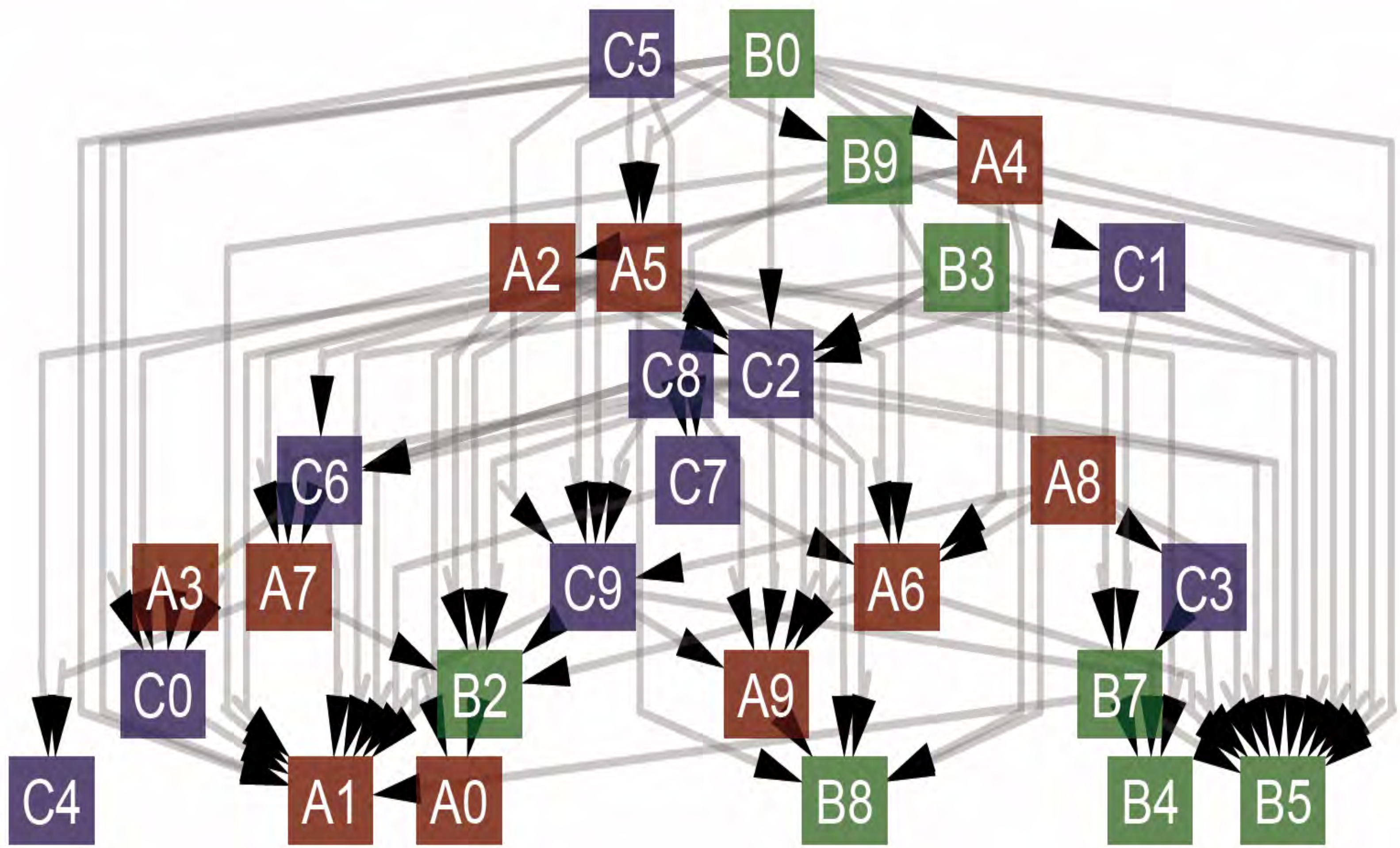
Complex sytems are hierarchical. Hierarchy is abundant, but no widely accepted quantitative interpretation of the origin and emergence of multi-level hierarchies exist

Motivation II   (Pigeons)

In a recent work we observed that during collective decision making, (i.e., navigating home as a single group), pigeons choose their common direction of flight as a result of dynamically changing hierarchical **leadership-followership interactions**.

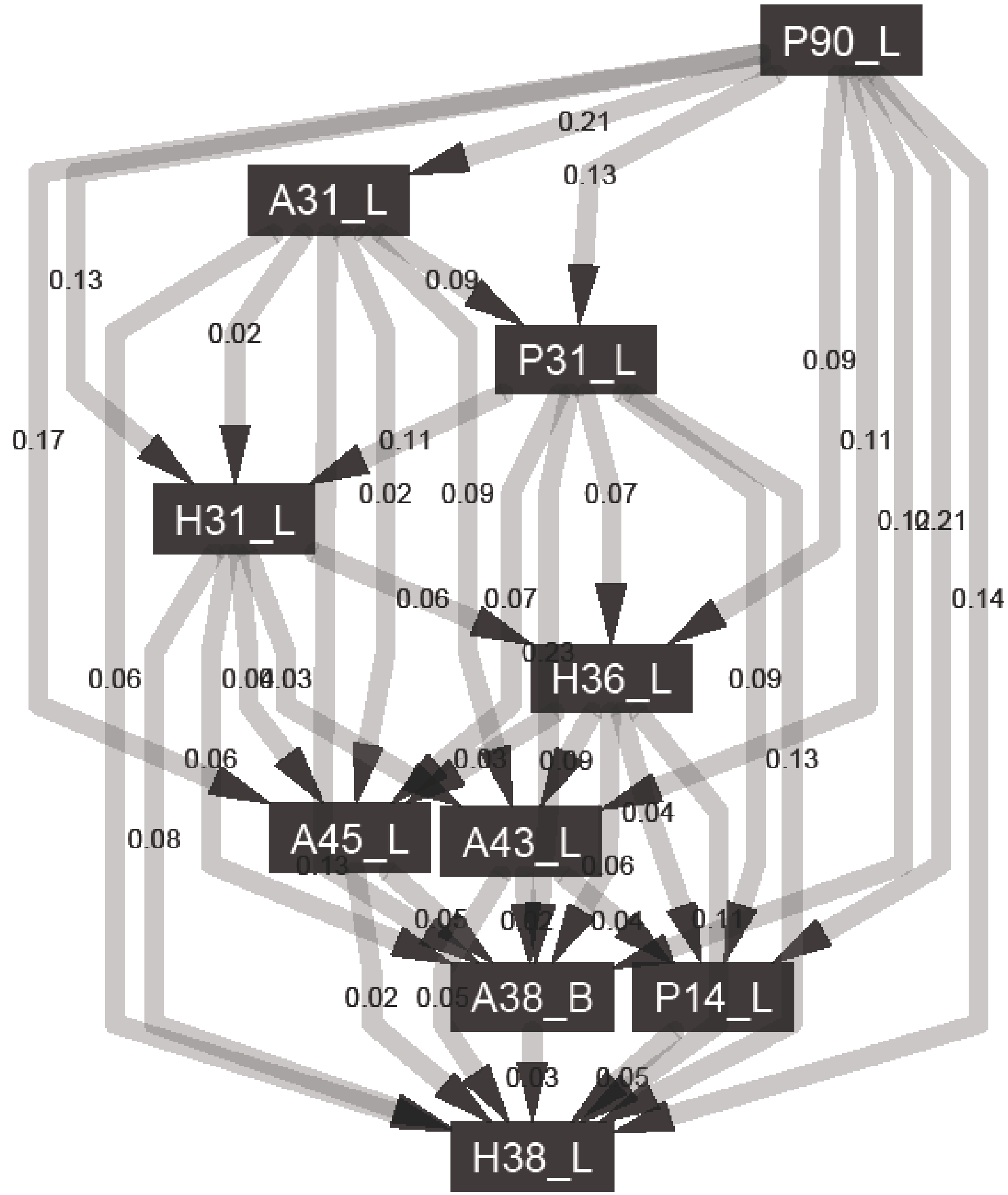
They also behave accoording to an order-hierarchy type **dominance hierarchy** in their loft


© 2011 Tracker development[1], Collective motion experiments[1,2]
[1] COLLMOT Research Project, Department of Biological Physics, Eötvös University
[2] OxNav Research Group, Department of Zoology, University of Oxford
2x speed
0 1 5 10 meters
0 5 10 25 feet


C5
B0
B9
A4
A2
A5
B3
C1
C8
C2
C6
C7
A8
A3
A7
C9
A6
C3
C0
B2
A9
B7
C4
A1
A0
B8
B4
B5

Digital video analysis of the moving pigeons around the feeding cup

Pairwise dominance graph as determined from „who is closer to the feeding cup"



„context dependent" (i.e., very different from navigational)

What are the main signatures of hierarchy (and such networks)?

Open question (order, embedded, flow, a mixture of these, measures of the level of hierarchy)

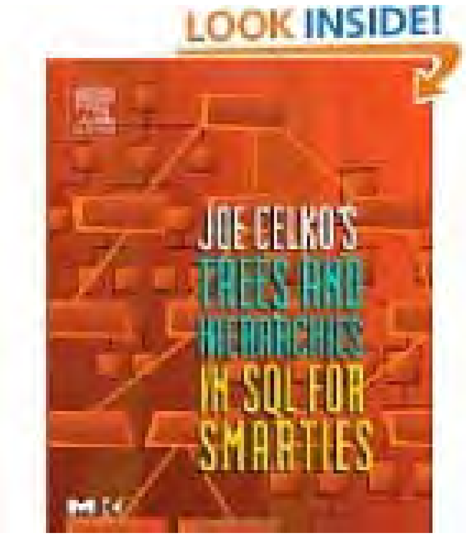
How do they emerge?

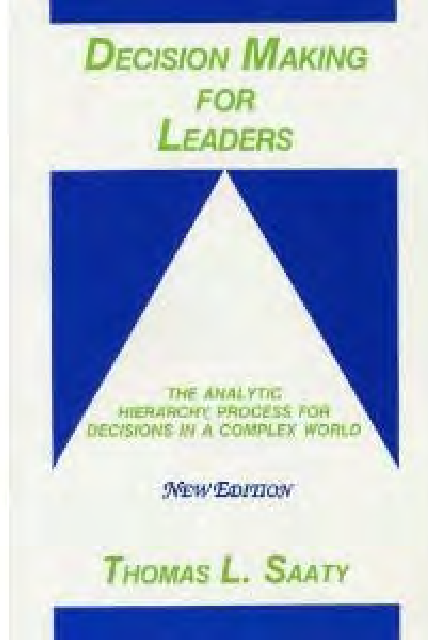
Open question

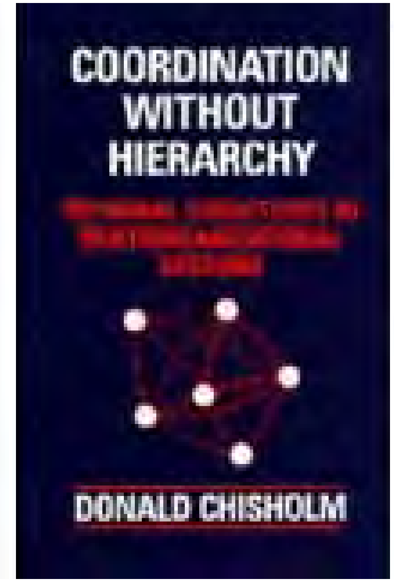
What are the main features optimized by hierarchy?

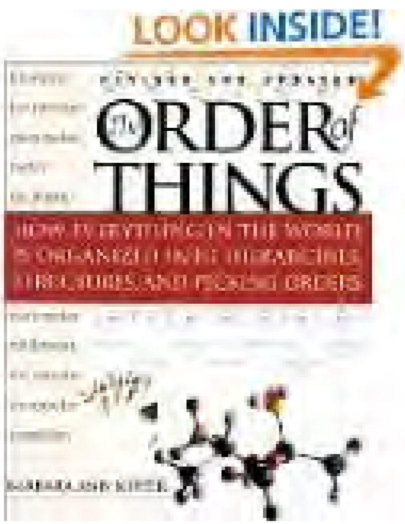
Flow of information? * More efficient production? Controllability??

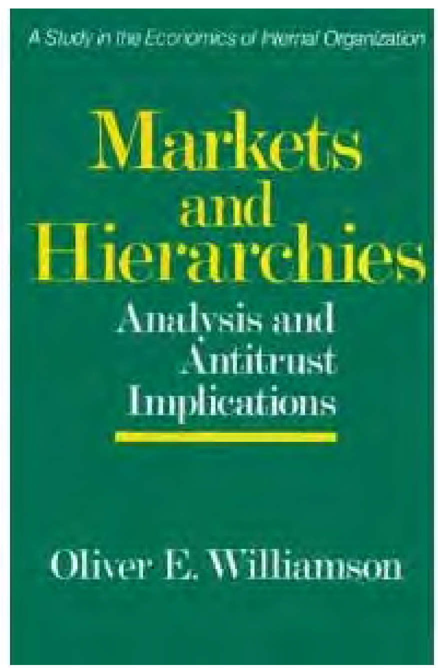
Better decision making process? *

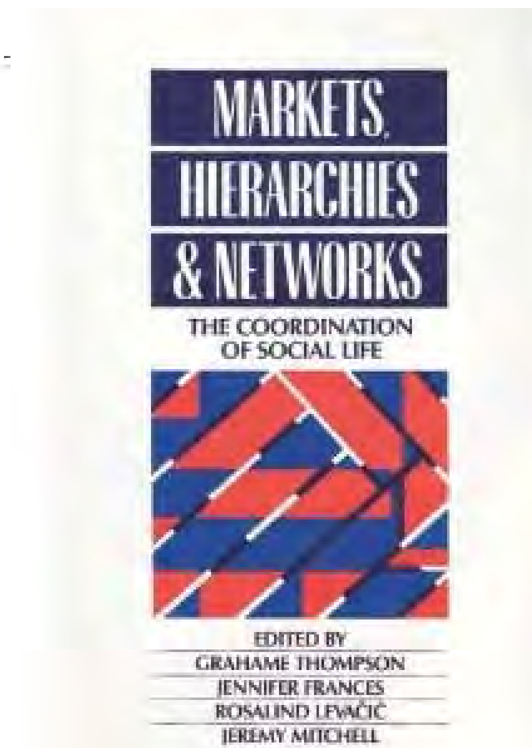

LOOK INSIDE!
JOE CELKO'S
TREES AND
HIERARCHIES
IN SQL FOR
SMARTIES


DECISION MAKING
FOR
LEADERS
THE ANALYTIC
HIERARCHY PROCESS FOR
DECISIONS IN A COMPLEX WORLD
NEW EDITION
THOMAS L. SAATY


COORDINATION
WITHOUT
HIERARCHY
DONALD CHISHOLM


LOOK INSIDE!
The ORDER of
THINGS


A Study in the Economics of Internal Organization
Markets
and
Hierarchies
Analysis and
Antitrust
Implications
Oliver E. Williamson


MARKETS,
HIERARCHIES
& NETWORKS
THE COORDINATION
OF SOCIAL LIFE
EDITED BY
GRAHAME THOMPSON
JENNIFER FRANCES
ROSALIND LEVAČIĆ
JEREMY MITCHELL

Hierachy Measure for Complex Networks

With E. Mones and L. Vicsek

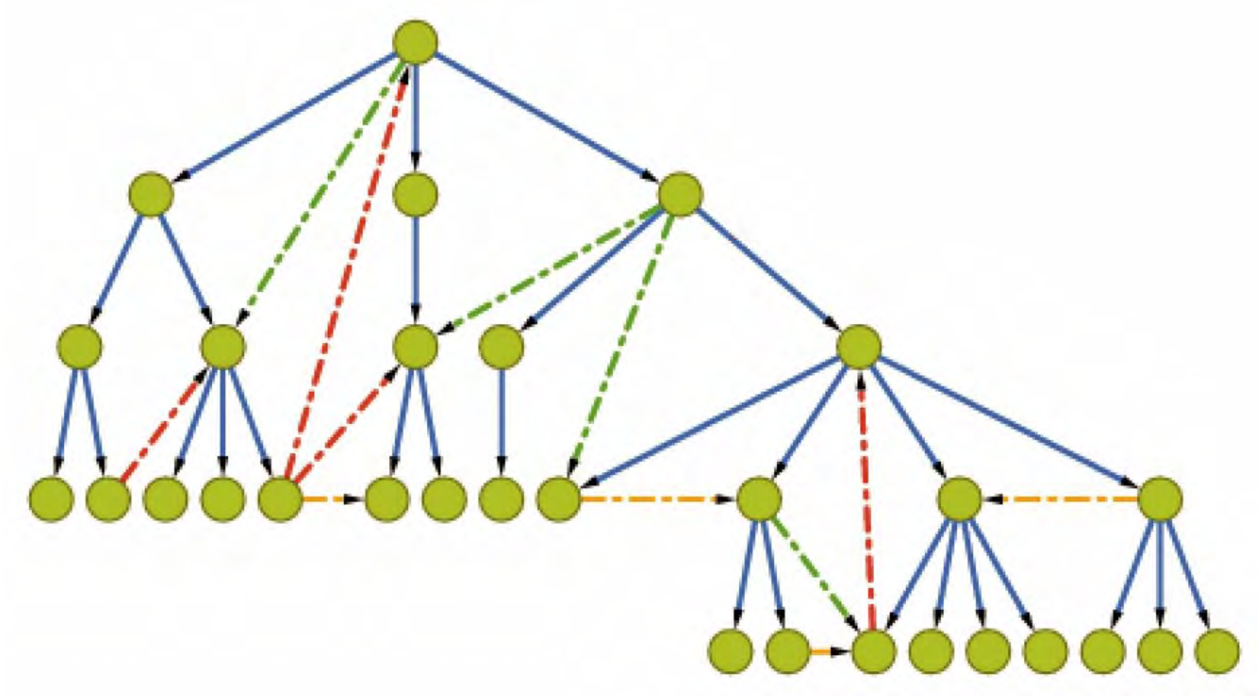
There exsists no widely applicable/accepted measure for the level of hiararchy

The measure we are looking for should not depend on any arbitrary parameters, or *a priori* metrics.

It should be useful for visualization purposes as well.

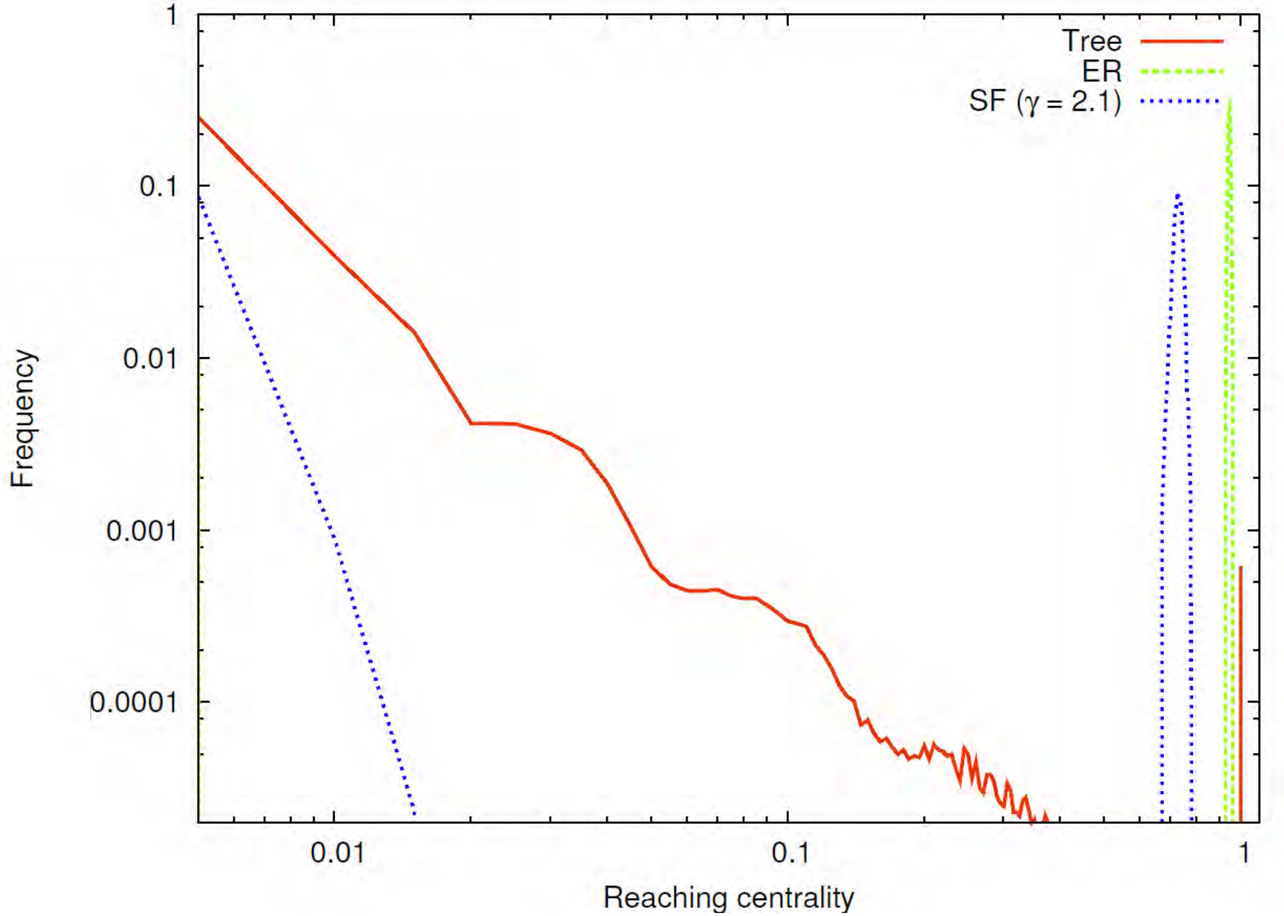
Local reaching centrality: The number of nodes reachable from the given node

Tests on a model graph with tunable hierachy



Start from tree and add edges so that a *p* percent of them points „downward”

Distribution of local reaching centralities



Local and global reaching centralities

Local: number of nodes accessible from node *i*

$$C_R(i) = \frac{1}{N-1} \sum_{j:0<d^{out}(i,j)<\infty} 1$$

$$C'_R(i) = \frac{1}{N-1} \sum_{j:0<d^{out}(i,j)<\infty} \left(\frac{\sum_{k=1}^{d^{out}(i,j)} \omega_i^{(k)}(j)}{d^{out}(i,j)} \right)$$

Global

$$H_R(G) = \sum_{i \in V} [C_R^{max} - C_R(i)]$$

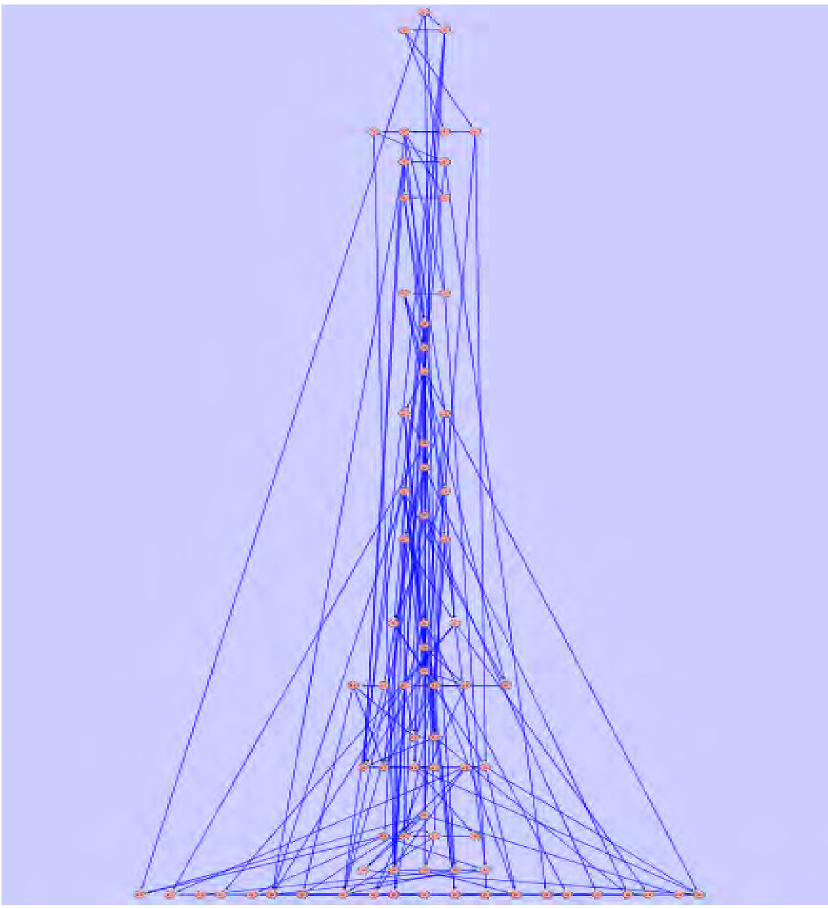
normalized

$$C_R = \frac{H_R(G)}{N-1}$$

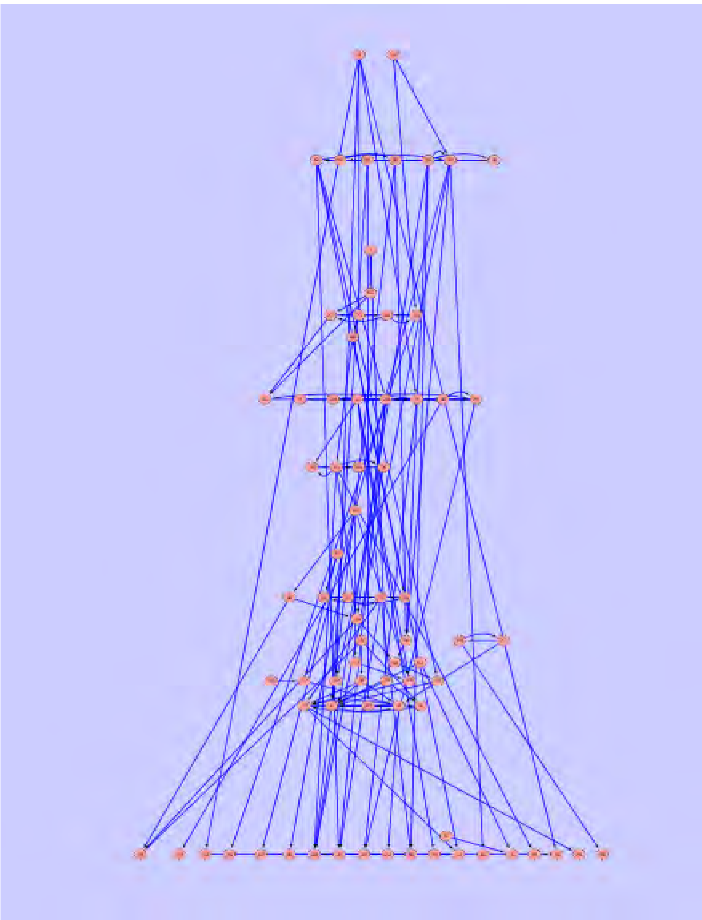
Graph	C_R	$\boldsymbol{\sigma}$
ER	0.058 ± 0.005	0.222 ± 0.010
SF ($\gamma = 2.1$)	0.207 ± 0.013	0.323 ± 0.009
Tree	0.997 ± 0.001	0.031 ± 0.004

Visualization of the hierarchical structure of a network with an intermediate level of hierarchy

(based on reaching centrality)



„Syntethic”



experiment

The case of optimal order hierarchy:

Group performance is maximized by hierarchical competence distribution

Our results were obtained by optimizing the group behaviour of the models we introduced through identifying the best performing distributions for both the competences (level of contribution to the best solution) and for the members' flexibilities/pliancies (willingness to comply with group mates).

Potential applications include choosing the best composition for a team, where „best" means better performance using the smallest possible amount of resources („competence costs money")

One of our main observations/statements is that most of the tasks to be completed during collective decision making can be reduced to an "estimation" (of the best solution) paradigm.

We consider the following general situation: finding the best solution happens in rounds of interactions during which

- each individual makes an estimation of the best solution based on its competence
(ranging from small to very good), and from the behaviours of its neighbours (neighbours being represented by nodes of various networks).

- the actual choice of the members also depends on their varying flexibilities (pliancies, i.e., the level to which they are willing to adopt the choices of their neighbours)

-**a collective "guess" about the true solution is made**

(then a new round starts)

Thus, we defined four generic Group Performance Maximization Models.

Because of the simplicity of our GPMMs, many real-life tasks can be mapped on each of them. The quality of the groups' performance, *Pe*, is quantifiable and characterized by a parameter with values in the [0, 1] interval.

Individual $_i$ has a competence level Co_i. Co_i also takes values from the [0, 1] interval. Each model/game consists of iterative steps. (i) The behaviour $Be_i(t+1)$ of agent (member) *i* at time step *t*+1, depends both on its own estimation $f(Co_i)$ regarding the correct solution, and on the (observable) average behaviour of its neighbours $j(\epsilon R)$ in the previous step *t*, $<Be_{tj}>j\epsilon R$:

$$Be_i(t+1) = (1-\lambda_i)\, f(Co_i) \text{ `+' } \lambda_i <Be(t)_j> \qquad j\epsilon R,$$

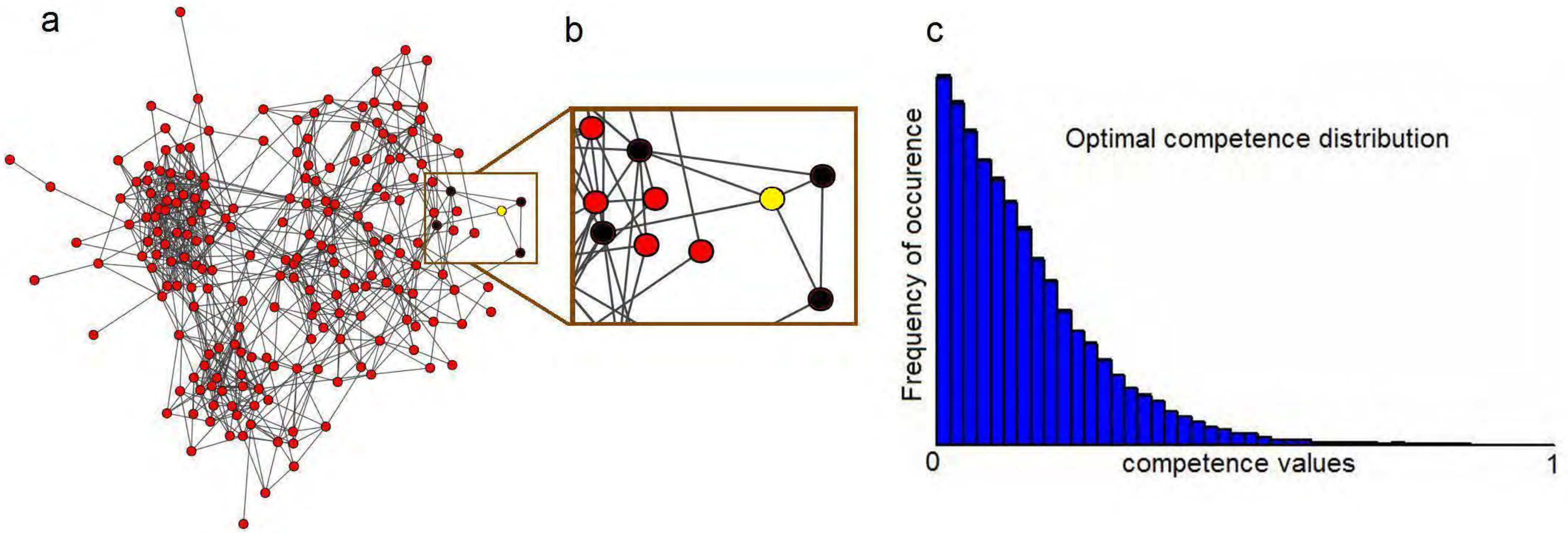
where '+' denotes "behaviour-dependent summation", and "behaviour" refers to various actions, such as estimating a value, casting a vote or turning into a direction, etc. The set of weight parameters λ_i takes values on the [0, 1] interval and defines the pliancy distribution.

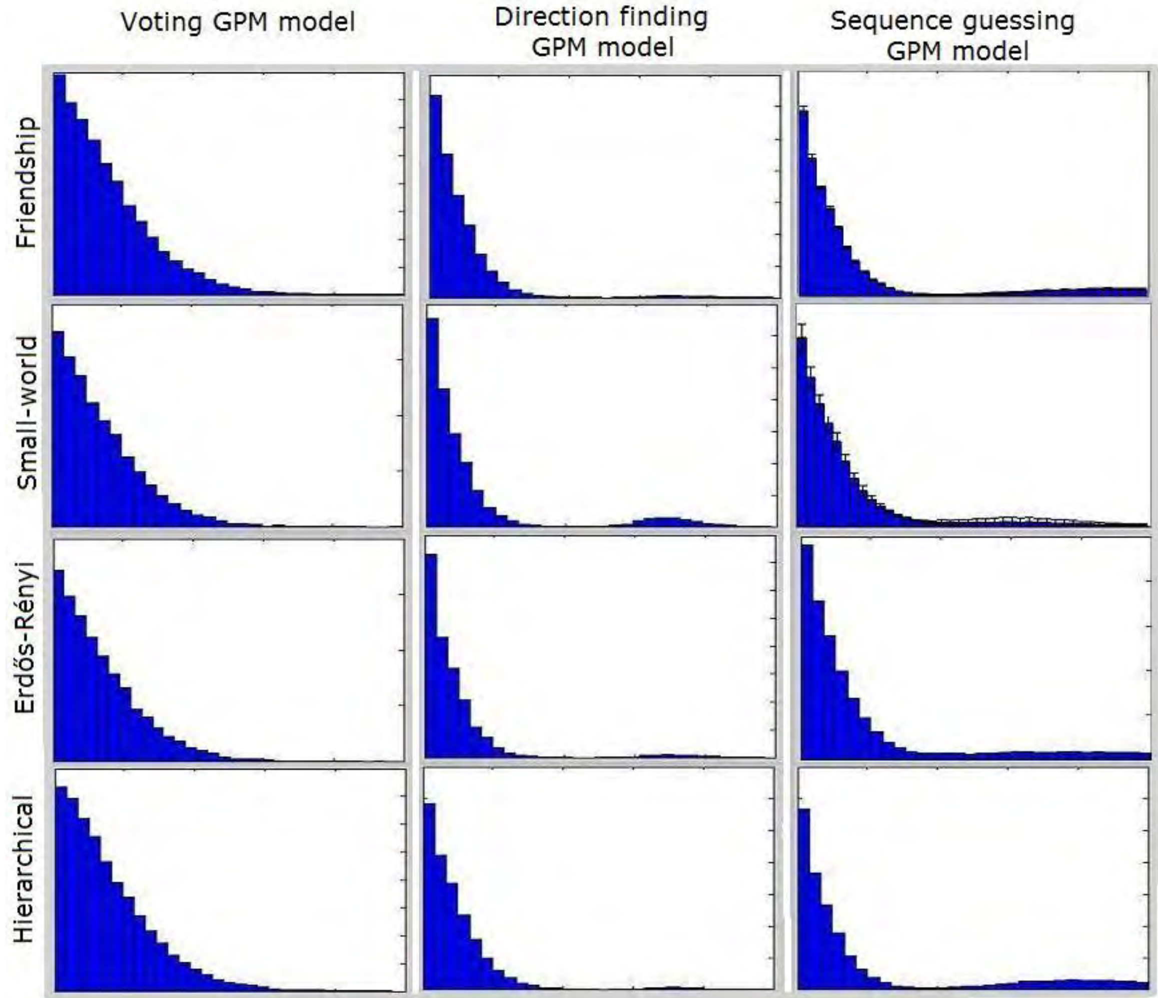
$$F = Pe - K<Co>,$$

Optimal distribution of competences is determined by maximizing the fitness *F* using a genetic algorithm

Voting model

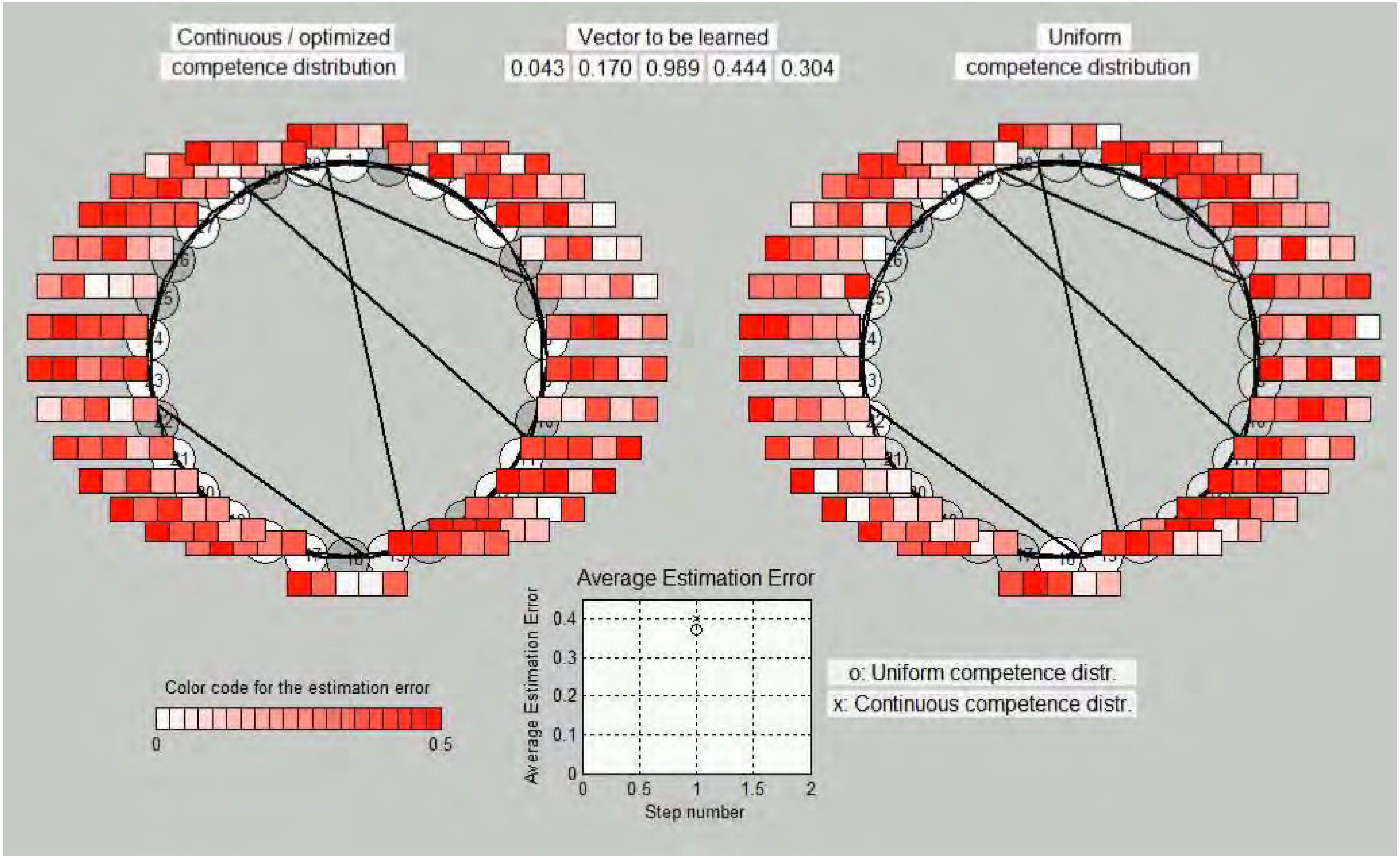
Simplest: guess whether up or down is the true state
ask nearest neighbours
take majority vote
make one more round



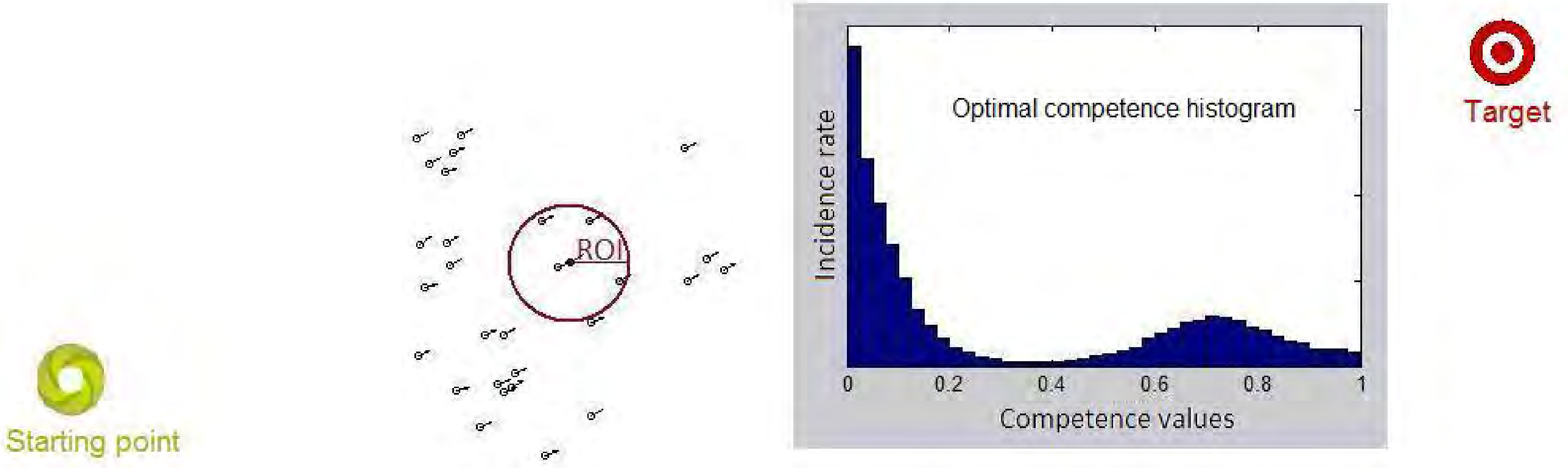


Voting GPM model
Direction finding GPM model
Sequence guessing GPM model
Friendship
Small-world
Erdős-Rényi
Hierarchical

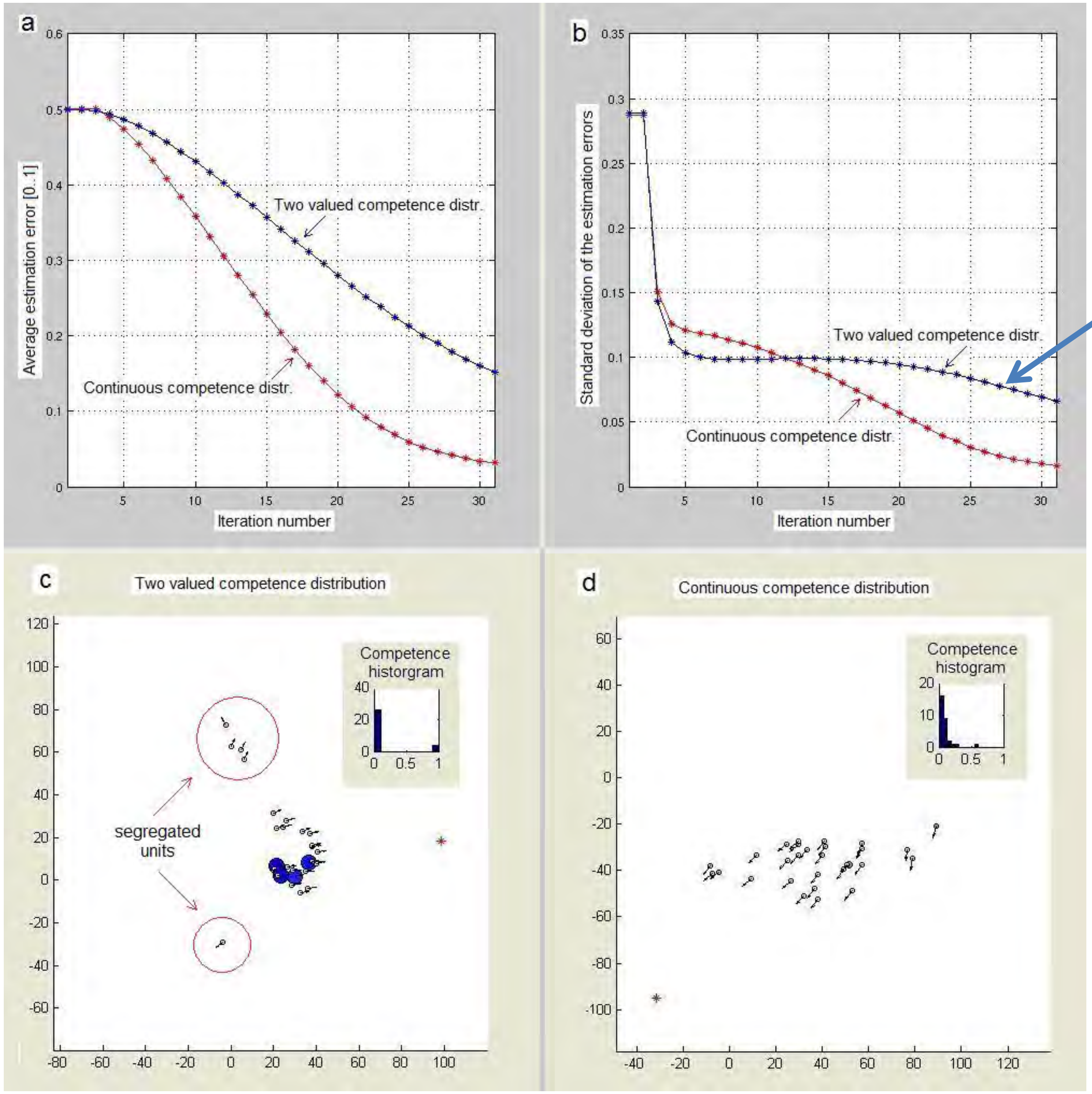
Number sequence guessing game on a small world graph



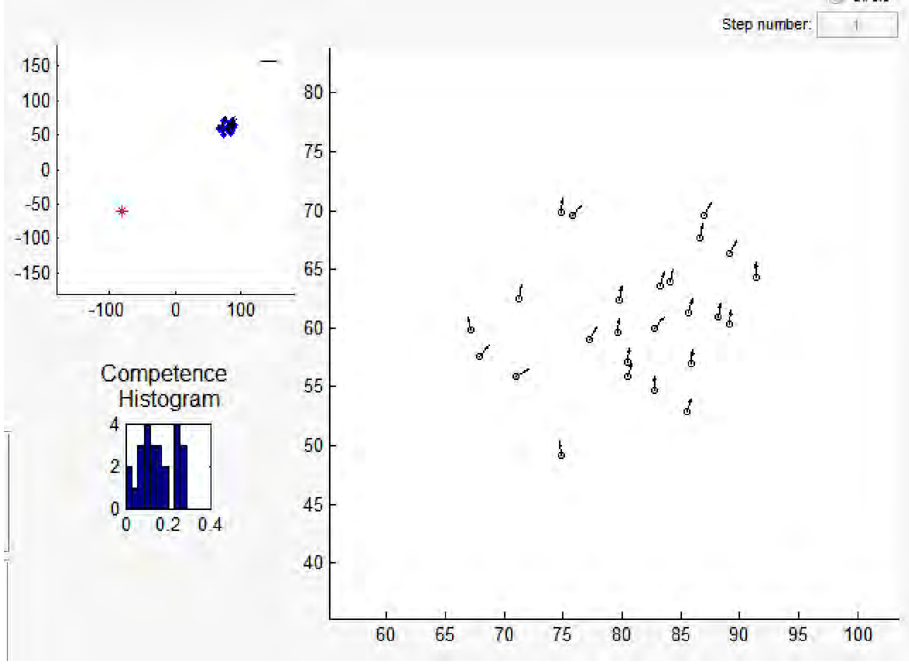
Homing pigeon flock model

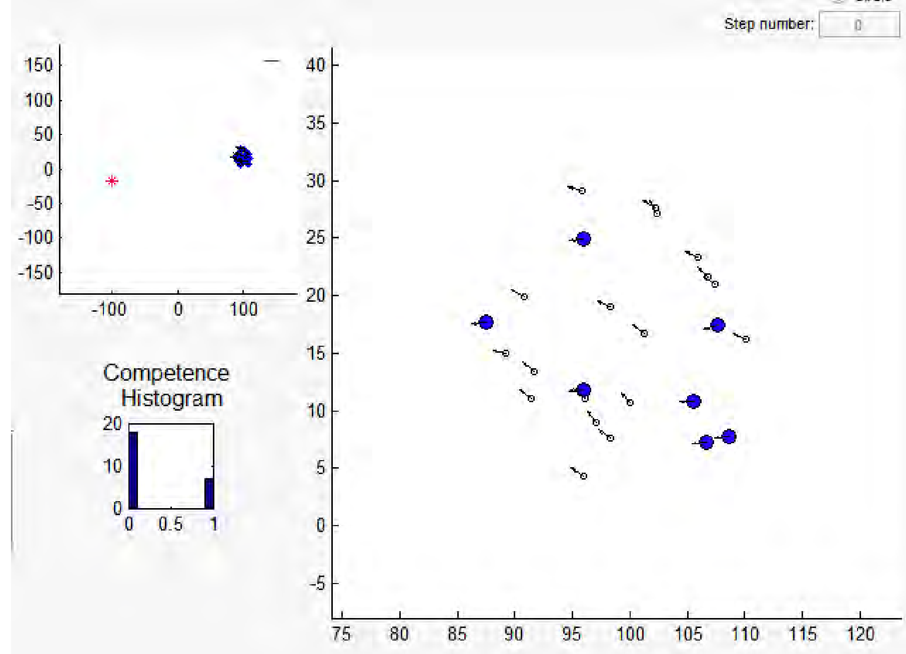


Interpretation: better mixing of the information



Sign of „segregation”


Step number:
Competence
Histogram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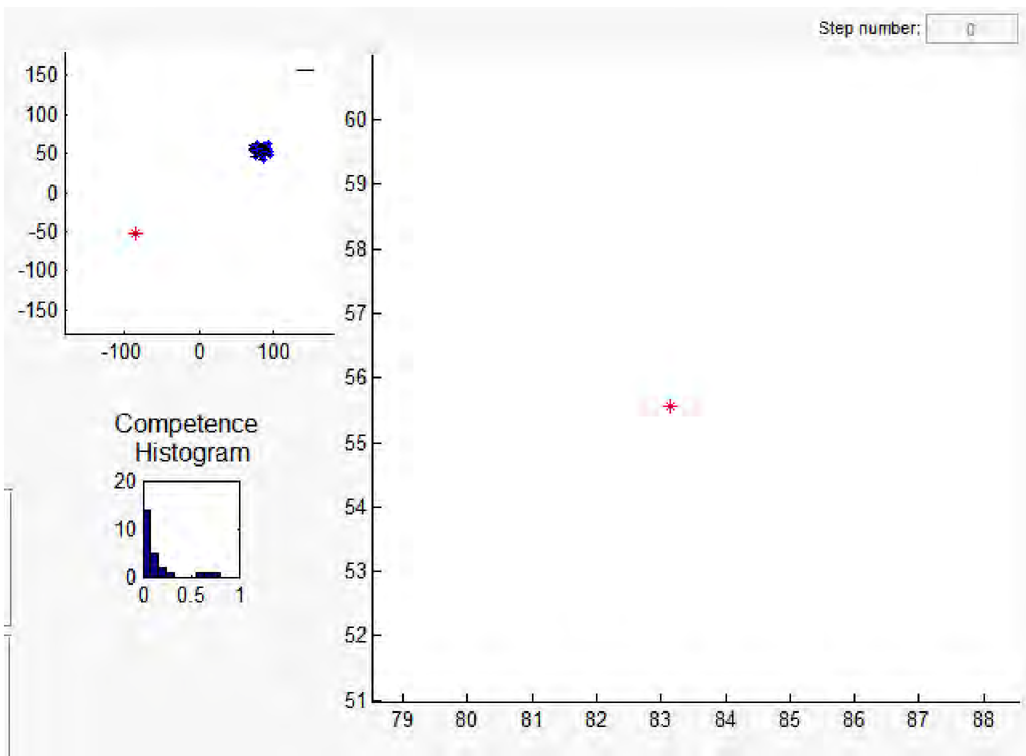
Step number:
Competence
Histogram