

Hierarchies in Humans & Opinion dynamics

Bioinspired Systems - 2022 Oct 6

Technical information

The presentations (pdf versions of the ppt files) and exam topics will be available on <https://hal.elte.hu/~lanna/>

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[Curriculum vitae in English and in Hungarian](#)

Anna Zafiris

Currently I am a research fellow at the Biological Physics Department of the Eotvos Lorand University of Science (Budapest, Hungary) in the group of Prof. Tamas Vicsek.
My main topic is Modeling the collective motion of animals.



Ph.D: Infobionics, 2008
Peter Pazmany University
Faculty of Information Technology

MSc (1) Physics, 2002
University of Sciences, Szeged, Hungary

MSc (2) Computational Mathematics, 2003
Orientation: Artificial Intelligence
University of Sciences, Szeged, Hungary

Contact Information:

ELTE, Biological Physics Department
Pazmany Peter stny. 1/A,
Budapest, Hungary, 1117

Room number: 3.79
Phone: + 36 3722-500/6361



New time?

Part 1

Hierarchies in Humans

(Continuation of the previous lecture)

Hierarchy in humans



Answers? Ideas?

Large-scale human hierarchies: from small groups to ultrasocieties

Problem:

- What enforced the transition from small, genetically related cooperative H-G groups to huge anonymous, hierarchically organized societies, typically organized as states, “ultrasocieties”?
 - small, „traditional” HG societies: kin selection + reciprocal altruism
 - Only 10-12,000 years ago (vs. 200,000 y)
- Neolithic transition (agriculture, animal domestication, settlement and transition to more complex societies)



Existing theories

- Many theories, but non of them completely satisfactory
- Mostly anthropological, historical approaches (qualitative)
- Quantitative approaches are rare (but existent)

Mostly agent-based models:

- Barceló and Castillo (eds) 2016: Simulating Prehistoric and Ancient Worlds (Computational Social Sciences). Springer, Cham, Switzerland
- Grinin and Korotayev (eds) 2014: History & Mathematics: Trends and Cycles. Uchitel, Volgograd
- Pumain and Reuillon 2017: Urban Dynamics and Simulation Models (Lecture Notes in Morphogenesis). Springer, Cham, Switzerland
- AB models combined with game theory
 - Boix 2015, Political Order and Inequality. Cambridge Univ. Press, New Jersey
 - Greif 2006. Institutions and the Path to the Modern Economy: Lessons from Medieval Trade. Cambridge Univ. Press, New York

A field of science in its infancy

Quantitative \leftrightarrow Qualitative approaches

General \leftrightarrow specific approaches

Are there laws at all?

- Monotone tendencies
 - The scale of societies constantly grow
 - The complexity of societies constantly grow
 - The number of languages constantly decreases
- Noise
- For a completely random process there are no monotone tendencies

Contradicts that history is “simply one damned thing after another.”

(The quote has been variously attributed to Winston Churchill, Edna St. Vincent Milay, Arnold Toynbee, or any one of a half-dozen other candidates.)

„Cliodynamics” – A field of science in its infancy

„Cliodynamics is a transdisciplinary area of research integrating historical macrosociology, cultural and social evolution, economic history/cliometrics, mathematical modeling of long-term social processes, and the construction and analysis of historical databases.”

nature

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Published: 02 July 2008

Arise 'cliodynamics'

[Peter Turchin](#)

[Nature](#) 454, 34–35 (2008) | [Cite this article](#)

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If we are to learn how to develop a healthy society, we must transform history into an analytical, predictive science, argues Peter Turchin. He has identified intriguing patterns across vastly different times and places.

- The book by Turchin (2003) [Historical Dynamics: Why States Rise and Fall](#). Princeton Univ. Press, New Jersey - offers one of the deepest analysis
- Turchin et al, [War, space, and the evolution of Old World complex societies](#), PNAS, 2013

The screenshot shows the eScholarship website interface. At the top, there's a navigation bar with links like 'eScholarship', 'Most Visited', 'eLibrary', 'Media', 'Lectures', 'LLRE', 'Research', and 'Other Bookmarks'. Below this is the eScholarship logo and the text 'University of California'. A search icon is on the right. The main content area is titled 'Cliodynamics' with 'UC Riverside' below it. There are 'Submit' and 'Manage' buttons on the right. A 'Menu' dropdown is visible. Below the title, there's a breadcrumb trail: 'eScholarship > UC Riverside > The Institute for Research on World-Systems > Cliodynamics'. An 'About' section follows, containing a paragraph about Cliodynamics as a transdisciplinary area of research, mentioning historical macrosociology, cultural and social evolution, economic history/cliometrics, mathematical modeling, and the construction and analysis of historical databases. It also mentions the journal 'Cliodynamics: The Journal of Quantitative History and Cultural Evolution' as an international, peer-reviewed publication, with a 'More' link at the end.

Simplified train of thought

- **They identify two main factors: warfare and multilevel selection**
 - **Small H-G societies:** Throughout most of human history, people lived in small-scale, mostly egalitarian societies.
 - **Warfare over resources:** These tribes often engaged in warfare with each other, over various resources.
 - **Selfishness vs. Group behavior:** Although selfish behavior can be beneficial for the individuals within a group, when groups intensively compete with each other (for example, during warfare), those groups that have more cooperative and less selfish members have the advantage. Thus, human **societies** are subject **to multilevel selection**.

The effects of warfare on social evolution:

- Groups become internally more cohesive
- Technological progress, including military and organizational applications
- „God always favors the big battalions” (Napoleon / Turenne) → **Enlargement of group sizes**

The **capacity of the human brain** has its limits,

- it cannot handle social relations in detail among more than around 150 people (Dunbar number).
- → there is a limit to the size of egalitarian, face-to-face human groups.

Simplified train of thought – cont.

Pressure on the group size to grow \leftrightarrow Dunbar no.

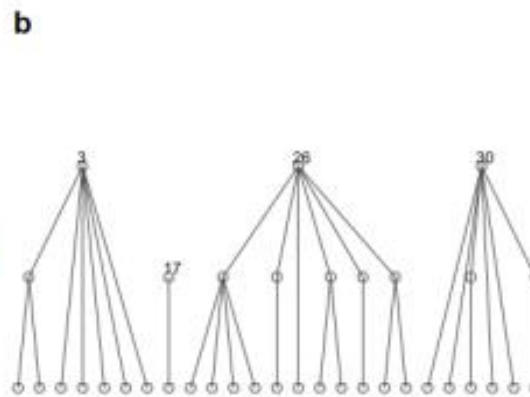
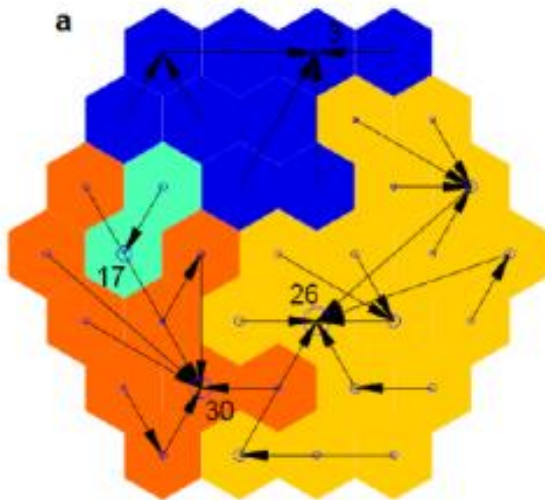
Assumption: the evolutionary response to this dilemma:

1. the ability to demarcate group membership based on cultural traits (language, dialect, clothing, etc.)
2. hierarchical organization, allowing group sizes to grow basically ad infinitum ((formal roles))

Each element within a given level of a strictly hierarchical system needs to have, at most, $n+1$ connections: n : „span of control”; $+1$: its superior

Turchin-model:

- Nodes stand for a political entity (e.g., villages)
- Numerical experiments with AB model:
 - The modelled area is divided into hexagonal cells (autonomous local communities „villages”)
 - Each of these villages are characterized by:
 - **a base-line resource level**, accounting for the heterogeneous environment, defining the productive/demographic potential of the region (a tunable parameter)
 - **actual resource level**, the base-line resource level minus the costs of the various actions in which the given community participates



A system of 37 communities organizing themselves into four polities.

The numbers in the hexagons mark the chief communities.

a. Spatial view.

b. The hierarchical structure

The Turchin-model in detail:

- Polities are organized in a hierarchical way
- Subordinate communities pay „tribute” to their superiors (a fixed portion of their total resources)
→ the total resource level of a community =
 = base resource level - tribute + the tribute it receives from its subordinates
- Polities may engage in warfare
 - Rebel
 - Conquest
 - Being attacked

Probability of warfare

A polity will attack its weakest neighbor if

- i. it estimates that the attack will be successful
- ii. it is ready to pay the corresponding costs and
- iii. it is not too devastated from previous wars.

Quantitatively, the probability of an attack is:

$$A_{ij} = P_{ij} \cdot e^{-\beta c_{ij}} \cdot \frac{F_i}{F_{i,0}}$$

P_{ij} : the probability of success

(an attack by community i on community j)

$$P_{ij} = \frac{F_i^a}{F_i^a + F_j^a}$$

F_i : the power of polity i

$F_{i,0}$: the maximum possible power of polity i

a : is the „success probability exponent“

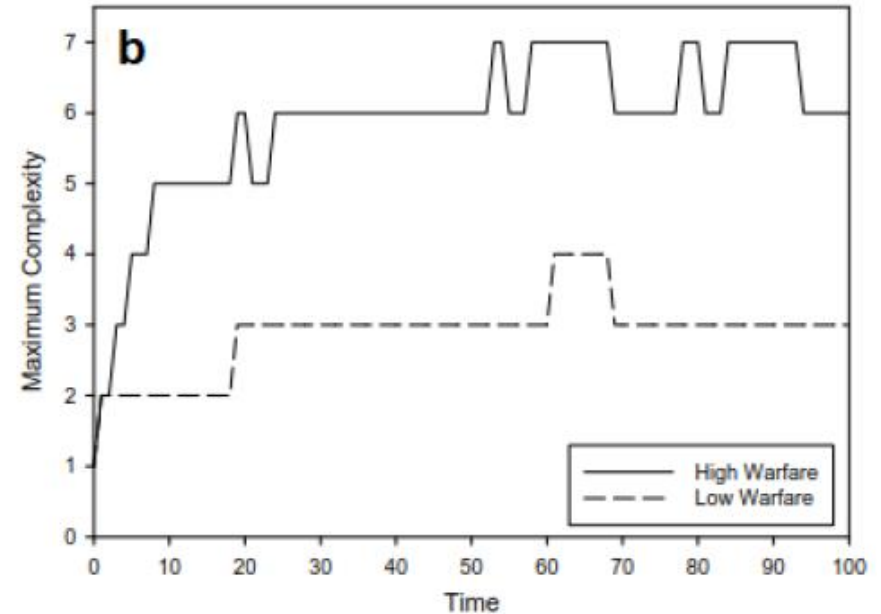
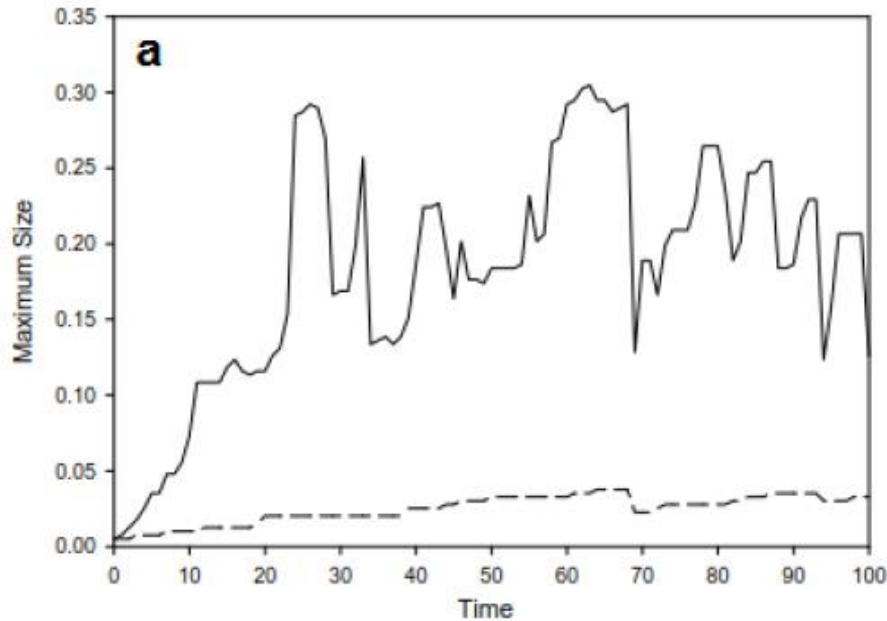
c_{ij} : cost of warfare

β : parameter

The Turchin-model in detail:

- Each time step is considered to be a year.
- Each year, the chief community decides whether to launch an attack on its weakest neighbor.
- If it decides to go to war:
 - it first attempts to conquer the bordering communities, followed by a series of „battles”, until it either suffers a defeat or the chief community of the victim polity falls.
 - Annexing the conquered communities may require restructuring the hierarchical organization of the winner polity (the number of max. subordinates is a parameter varying between 4 and 10)
 - the direct subordinates of the aggressor chief community might decide to **secede** if they estimate that the attack will be unsuccessful.
 - spatial separation from the master state, together with all the subordinate communities of the rebelling village

Results



- (a) The size and
- (b) the hierarchical complexity of the polities under low and high pressure of war.

Intense warfare results in larger and more complex polities.

Provides a fission-fusion cycle reminiscent of the dynamics characterising early states of humans.

The model with realistic historical data

- A more detailed version
- **Afroeurasian** landmass divided into a **grid of 100 × 100-km squares**
- **Grid cells are characterized** by existence of agriculture, biome (e.g., desert), and elevation
- **At the beginning of the simulation**, each agricultural square is inhabited by an **independent polity**
- Cells adjacent to the steppe are “**seeded**” with **military technology** (MilTech) traits, which gradually diffuse out to the rest of the landmass
- Each cell is inhabited by a community that has a “**cultural genome**,” a vector taking values of 1 or 0, depending on whether an ultrasocial trait is present.
 - such traits are costly: the probability of losing it is big, thus, in the absence of other evolutionary forces, they are present in the landscape at a very low frequency. The force that favors their spread is warfare
- Agricultural cells can conquer other such squares, building multicell polities. **The probability of winning** depends on relative powers, determined by the **polity size** (number of cells) and the average **number of ultrasocial traits**.
- The losing cell may copy the cultural genome of the victor.

causal chain: spread of military technologies → intensification of warfare → evolution of ultrasocial traits → rise of large-scale societies

Data

- polities that controlled territories greater than $\sim 100,000$ km² between 1,500 BCE and 1,500 CE
- on the Afroeurasian landmass
- by taking 100-year time windows, *imperial density maps* indicating the frequency and distribution of large-scale societies
- 7,941 empirical points

- predicting where and when the largest-scale complex societies arise
- hotspots appear in Mesopotamia, Egypt, and North China
- near the steppe frontier, where MilTech diffuse first, tipping the selection in favor of ultrasocial traits

Part 2 - Opinion dynamics



Opinion dynamics

- The scientific field aiming to understand the way „opinions” spread in human communities.
 - The community is usually described by means of **networks**
 - Nodes are individuals
 - Links are the ties (connections)
 - Direction
 - Strength
 - „opinion” (or „state” of the node) is usually described by a scalar (binary or continuous)
- Close relation to fields studying other spreading phenomena
 - Infection spreading



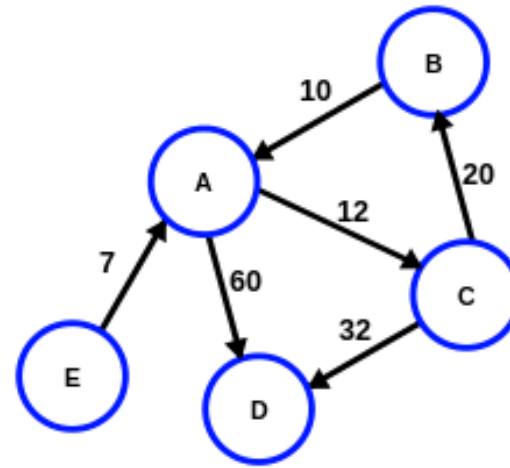
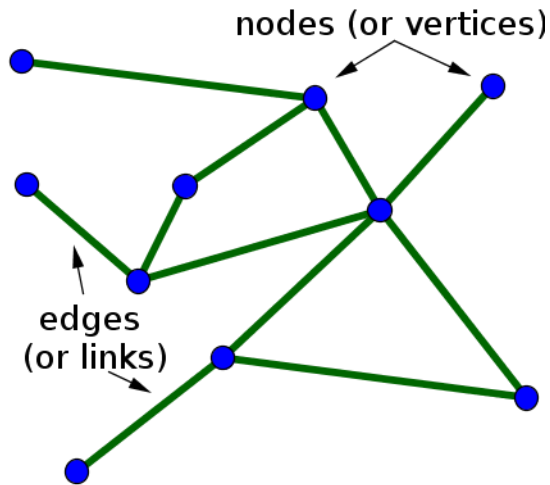
- Relevant **general questions** include:
 - What are the fundamental interaction mechanisms („local rules”) that allow for the emergence of
 - Consensus / polarization / fragmentation
 - a shared culture
 - a common language, etc. ...
 - What favors the homogenization process? What hinders it?
- “Unfortunately”: Opinion formation is a **complex process** affected by the interplay of different elements, including the
 - Individual predisposition / family background
 - Background knowledge
 - External information (e.g. public media)
 - Etc.

Typical models

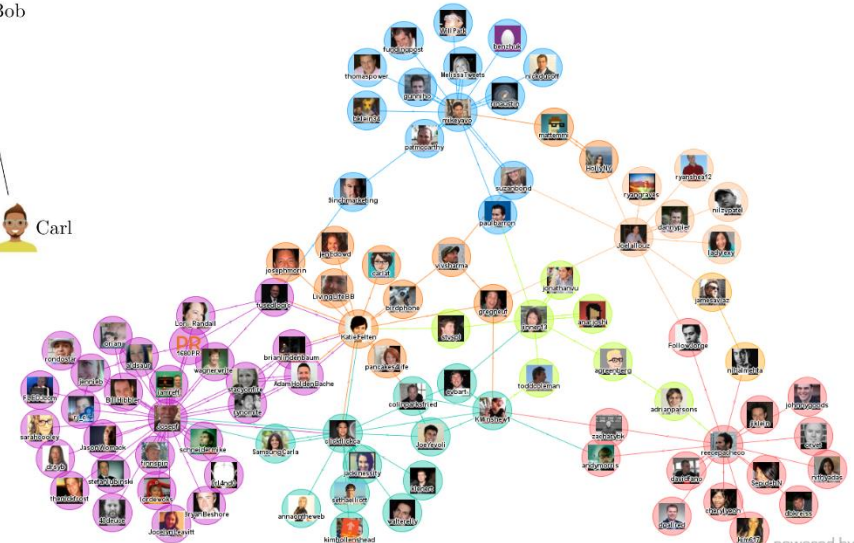
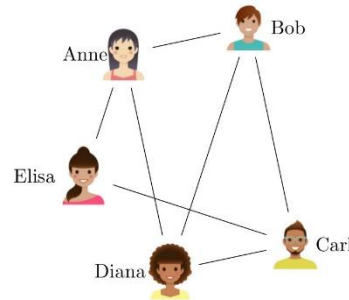
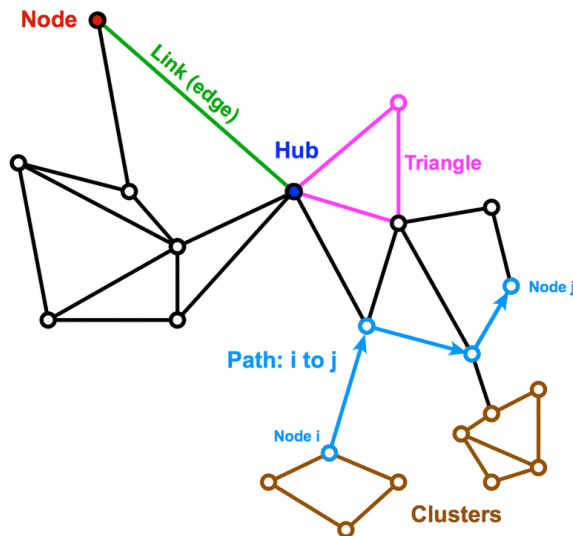
- Consider a finite number of connected agents
- each possessing opinions, described by variables,
- Assume certain *local rules* by which opinions change
 - Change of opinions result from interactions, either with peers or other sources.
- Opinions:
 - Variables:
 - one dimensional/multidimensional vector
 - discrete (the components can assume a finite number of states)
 - or continuous (values in the domain of real numbers)
- Connections:
 - Topology of the interaction NW (what is realistic?)
 - “Heritage” from physics: lattices or all-to-all (MF);
(hardly realistic in social context)

- Drawbacks of the models:
 - many simplifications;
 - many of the omitted parameters (most probably) have a fundamental effect in the final dynamics
 - Hard to say when are the results „good” (polarization)
- (It is said to have) Success in:
 - Agreement
 - Cluster formation
 - Transition between order (consensus) and disorder (fragmentation)

Basic concepts of networks



With some network analysis



Binary opinions

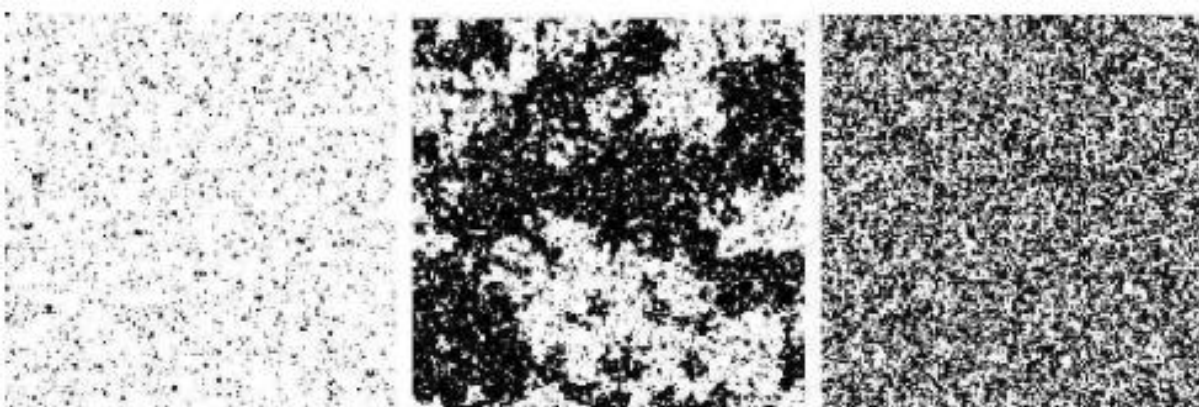
- Discrete, one dimensional
- 0/1; yes/no; etc
- Interpretation in op. dyn: political questions
infection models: infected / not
market behavior: selling/buying



Very first opinion dynamic model by physicist:
1971, Weidlich

Ising model metaphor

- Consider a collection of N spins (agents): s_i
- They can assume two values: ± 1
- Each spin is energetically pushed to be aligned with its nearest neighbors.
- The total energy is:
(the sum runs on the pairs of nearest-neighbors)
$$H = -\frac{1}{2} \sum_{\langle i,j \rangle} s_i s_j$$
- Elementary move:
- a single spin flip is accepted with probability $\exp(-\Delta E/k_B T)$
 - ΔE : change in the energy
 - T : temperature (In ferromagnetic systems thermal noise injects fluctuations – tends to destroy order)
 - Critical temperature T_c : above: the system is macroscopically disordered
under: long-range order is established



Snapshots of equilibrium configurations of the Ising model (from left to right) below, at and above T_c .

Relation to opinion dynamics models

- Each agent has one opinion represented as a spin:
a choice between two options
- Spin couplings: peer interactions (social conformity)
- Magnetic field: external information / propaganda
- Simple, but attractive model

Potts model (1951)

- a generalization of the Ising model
- Each spin can assume one out of q values
- equal nearest neighbor values are energetically favored.
- The Ising model corresponds to the special case $q=2$ ²⁸

Voter model

- Originally introduced to analyze competition among species, early 1970s
- Rather crude description of any real process
- Popular: it is one of the very few non-equilibrium stochastic processes that can be solved exactly in any dimension
- its name stems from its application to electoral competitions

- **The model:**

- each agent in a population of N holds one of two discrete opinions:

$$s = +/- 1$$

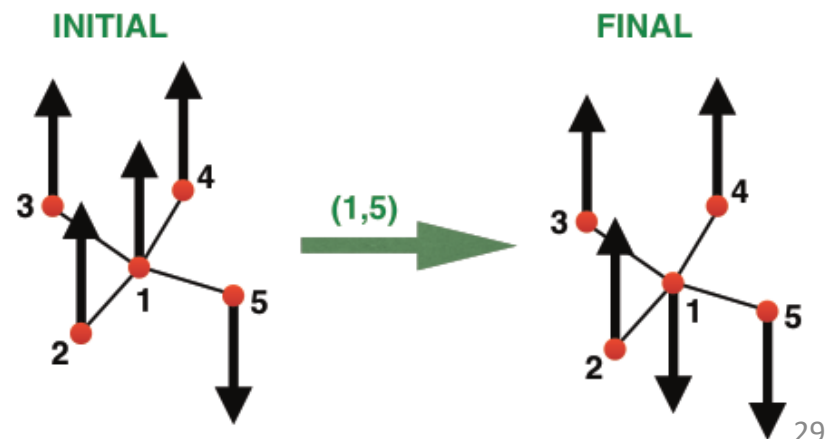
- agents are connected by an underlying **graph** (topology)

- **At each time step:**

a random agent i is selected

(1) along with one of its neighbors j (5) and the agent takes the opinion of the neighbor: $s_i = s_j$

(alignment *not* to the majority, but to a random neighbor)



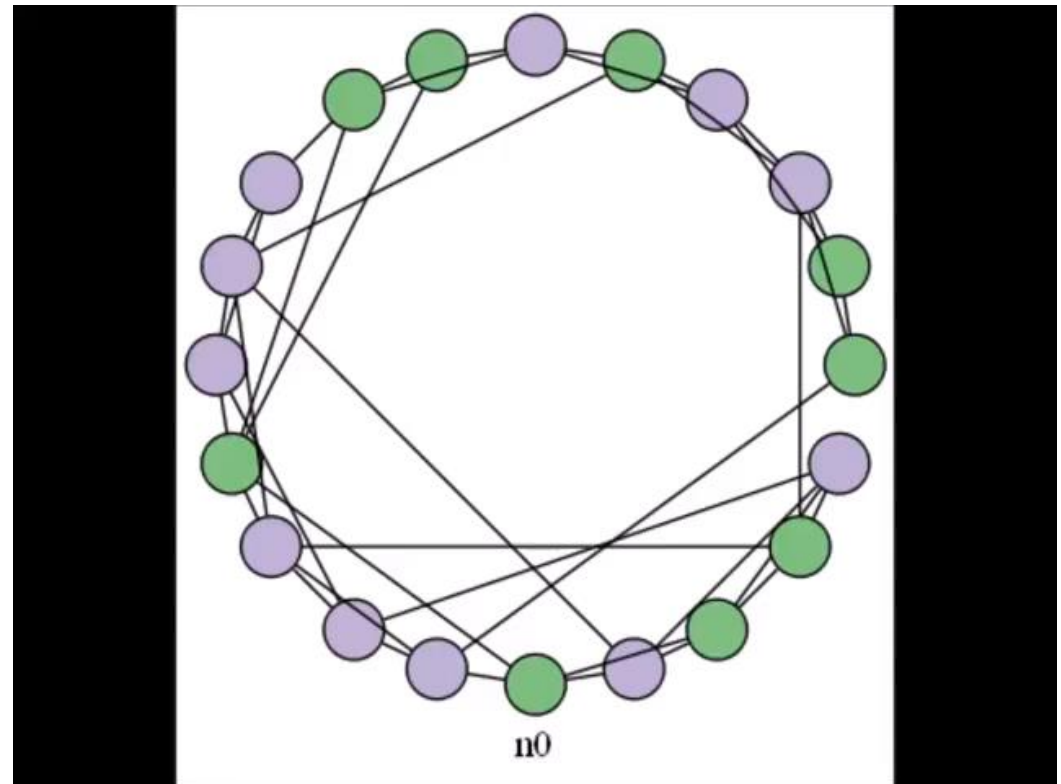
Behavior of the Voter model

- Has been extensively studied
- If people are modeled as vertices in a d -dimensional hyper-cubic lattice.
 - For finite system: for any dimension d of the lattice, the voter dynamics **always leads to** one of the two possible **consensus** states: each agent with the same opinion $s = 1$ or $s = -1$.
 - The probability of reaching one or the other state depends on the initial state of the population.
 - Time needed for reaching the consensus state:
 - $d = 1: T_N \sim N^2$
 - $d = 2: T_N \sim N \ln N$
 - $d > 2: T_N \sim N$
 - For infinite systems: consensus is reached only if $d \leq 2$

Extensions of the voter model

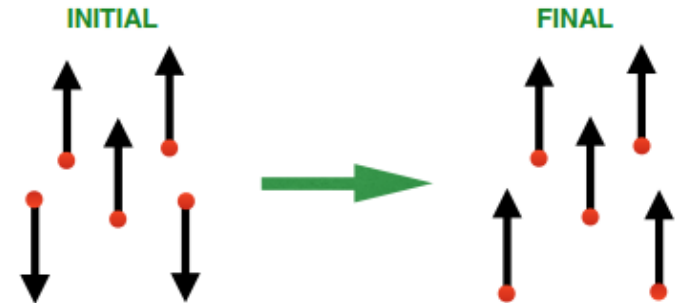
- Introduction of “zealots”: individuals who do not change their opinion
- Constrained voter model:
 - agents can be leftist, rightist, centralist;
 - Extremists do not talk to each other (discrete analogue of the bounded confidence model)
- Communication is based on various NW

Voter model on a small world network
<https://www.youtube.com/watch?v=VmhSTdrsimek>



Majority rule model

- Motivation: describing public debates
- (Galam, 2002)
- Definition:
 - Population of N agents
 - A fraction p_+ of agents has opinion +1
 - $p_- = 1 - p_+$ has opinion -1
 - Everybody can communicate with everybody else
 - At each interaction:
 - A group of r agents are selected at random (“discussion group”)
 - Consequence of this interaction: each agents take the majority opinion inside the group
 - r is taken from a given distribution at each step
 - If r is odd: there is always a clear majority
 - If r is even: in case of tie: a bias is introduced in favor of one of the options (Inspired by the principle of “Social inertia” holding that people are reluctant to accept a reform if there is no clear majority in its favor)



Basic features of the MR model

- Original definition:
 - There is a *threshold fraction* p_c such that if $p_o^+ > p_c$, then all agents will have opinion +1 in the long run
 - Time needed for the consensus: $T_N \sim \log N$
 - If the group sizes r are odd: $p_c(r) = 1/2$ (due to the symmetry)
 - If they can be even too: $p_c < 1/2$, that is, the favored opinion will eventually win, even if it was originally in minority
- For fixed odd r , group size & mean field approach: analytically solvable for both finite N and for $N \rightarrow \infty$
- Many variants and modifications

Social impact theory

- Bibb Latané (psychologist), 1981:
- **social impact**: any influence on individual feelings, thoughts or behavior that is created from the real, implied or imagined presence or actions of others.
(„Collective” behavior)
- The impact of a social group on a subject depends on:
 - The number of individuals within the group
 - Their convicting power
 - Their distance from the subject (in an abstract space of personal relationships)
- Originally a cellular automata was introduced by Latané (1981) and later refined by Nowak et al (1990).

Social impact theory – the model

- A population of N individuals
- Each individual i is characterized by
 - an opinion $\sigma_i = \pm 1$
 - Persuasiveness p_i : the capability to convince someone **to change** opinion (a real value)
 - Supportiveness: s_i : the capability to convince someone **to keep** its opinion (a real value)
(these are assumed to be random)
- The distance between agents i and j d_{ij} ,
- $\alpha > 2$ parameter defining the how fast the impact decreases with the distance

$$I_i = \underbrace{\left[\sum_{j=1}^N \frac{p_j}{d_{ij}^\alpha} (1 - \sigma_i \sigma_j) \right]}_{\text{Persuasive impact (to change)}} - \underbrace{\left[\sum_{j=1}^N \frac{s_j}{d_{ij}^\alpha} (1 + \sigma_i \sigma_j) \right]}_{\text{supportive impact (to keep opinion)}}$$

Persuasive impact (to change)

supportive impact (to keep opinion)

Opinion dynamics:
$$\sigma_i(t+1) = -\text{sgn}[\sigma_i(t)I_i(t) + h_i]$$

h_i : personal preference, originating from other sources (e.g. mass media)

a spin flips if the pressure in favor of the opinion change overcomes the pressure to keep the current opinion ($I_i > 0$ for vanishing h_i)

General behavior of the social impact model

- In the absence of individual fields (personal preferences):
 - the dynamics leads to the dominance of one opinion over the other, but not to complete consensus.
- In the presence of individual fields:
 - these minority domains become metastable: they remain stationary for a very long time, then they suddenly shrink to smaller clusters, which again persist for a very long time, before shrinking again, and so on (“staircase dynamics”).

- Many modification / extensions:
 - Learning
 - Presence of a strong leader
 - Etc.

Schweitzer and Holyst included: (2000)

- Memory: reflecting past experience
- A finite velocity for the exchange of information between agents
- A physical space, where agents move.

Continuous opinions

- In many cases more realistic
- Requires **different framework**
 - Concepts like “majority” or “opinion equality” don’t work
 - Has a different ‘history’
- **First studies** (end of 1970’s and 80’s):
 - Aimed to study the conditions under which a panel of experts would reach a common decision (“consensus”)
 - By applied mathematicians
- **Typically:**
 - **Initial state:** population of N agents with randomly assigned opinions, represented by real values within some interval.
discrete op. dyn. \leftrightarrow all agents start with different opinions
 - **Possible scenarios:** more complex
 - Opinion clusters emerging in the final stationary state:
 - one cluster: consensus,
 - two clusters: polarization
 - more clusters: fragmentation

Bounded confidence (BC) models

- **In principle:** each agent can interact with every other
- **In practice:** (often) there is a real discussion only if the opinions are sufficiently close:

bounded confidence

- **In the literature:** introducing a real number ε :
“*uncertainty*” or “*tolerance*”, such that:
- An agent with opinion x , only interacts with those whose opinion lies in the interval $]x-\varepsilon, x+\varepsilon[$
- („Homophily”)

Deffuant model

- population of N agents
- nodes of a graph: agents may discuss with each other if they are connected.
- Initially: each agent i is given an opinion x_i randomly chosen from the interval $[0, 1]$.
- **Dynamics:**
 - random binary encounters, i.e., at each time step, a randomly selected agent discusses with one of its neighbors, also chosen at random.
 - Let i and j be the pair of interacting agents at time t , with opinions $x_i(t)$ and $x_j(t)$
 - if the difference of the opinions $x_i(t)$ and $x_j(t)$ exceeds the threshold ε , nothing happens
 - If $|x_i(t) - x_j(t)| < \varepsilon$, then
 - μ : convergence param.
(μ in $[0, 1/2]$)

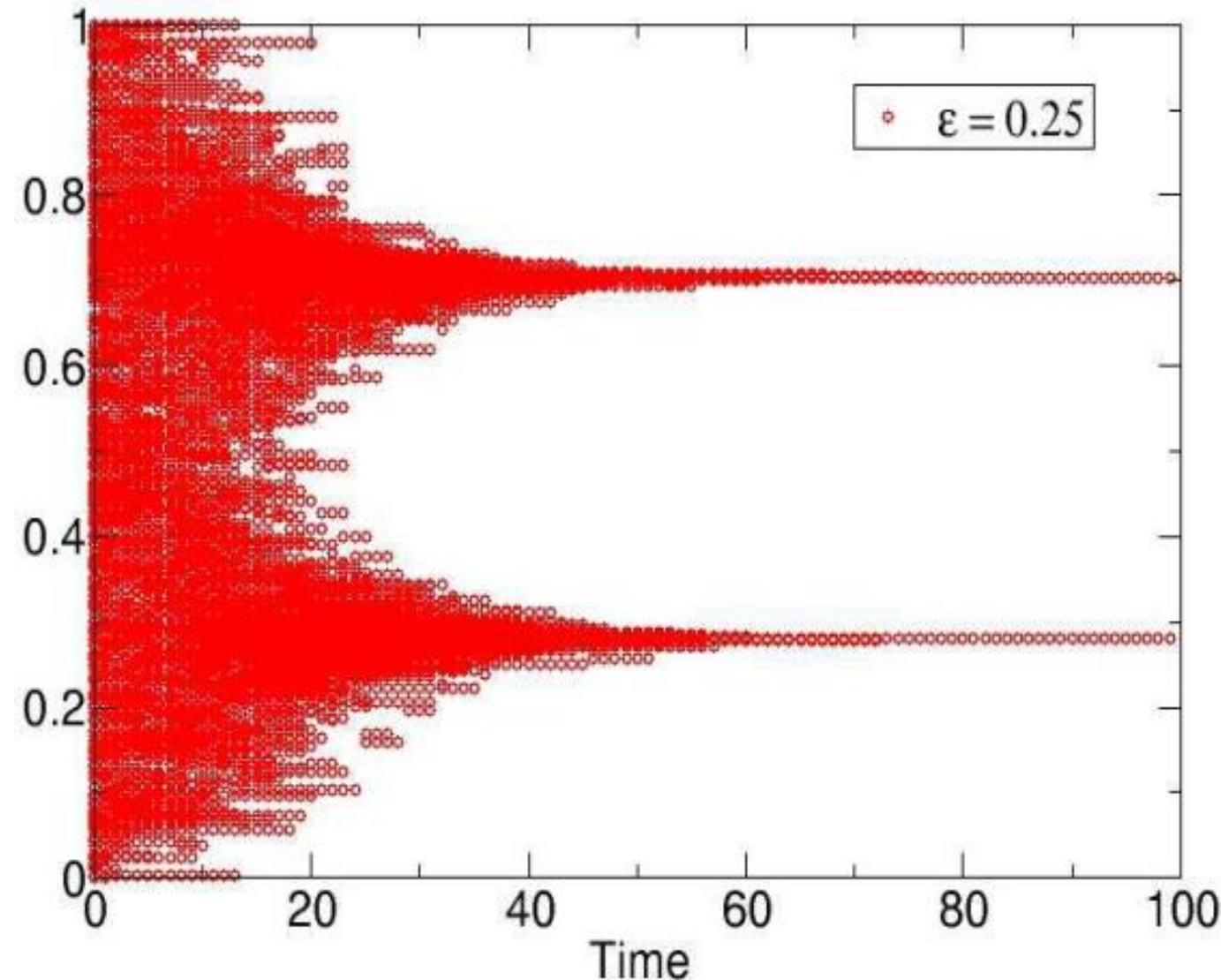
$$x_i(t+1) = x_i(t) + \mu[x_j(t) - x_i(t)]$$

$$x_j(t+1) = x_j(t) + \mu[x_i(t) - x_j(t)]$$

Behavior of the Deffuant model

- For any value of ε and μ , the average opinion of the agents' pair is the same before and after the interaction \rightarrow the global average opinion ($1/2$) of the population is invariant
- Patches appear with increasing density of agents
- Once each cluster is sufficiently far from the others (the difference of opinions in distinct clusters exceeds the threshold):
 - only agents *inside* the same cluster interact
 - the dynamics leads to the convergence of the opinions of all agents in the cluster
- In general:
 - the *number and size of the clusters* depend on the threshold ε (if ε is small, more clusters emerge)
 - the parameter μ affects the convergence time
 - (when μ is small, the final cluster configuration also depends on μ)

Behavior of the Deffuant model

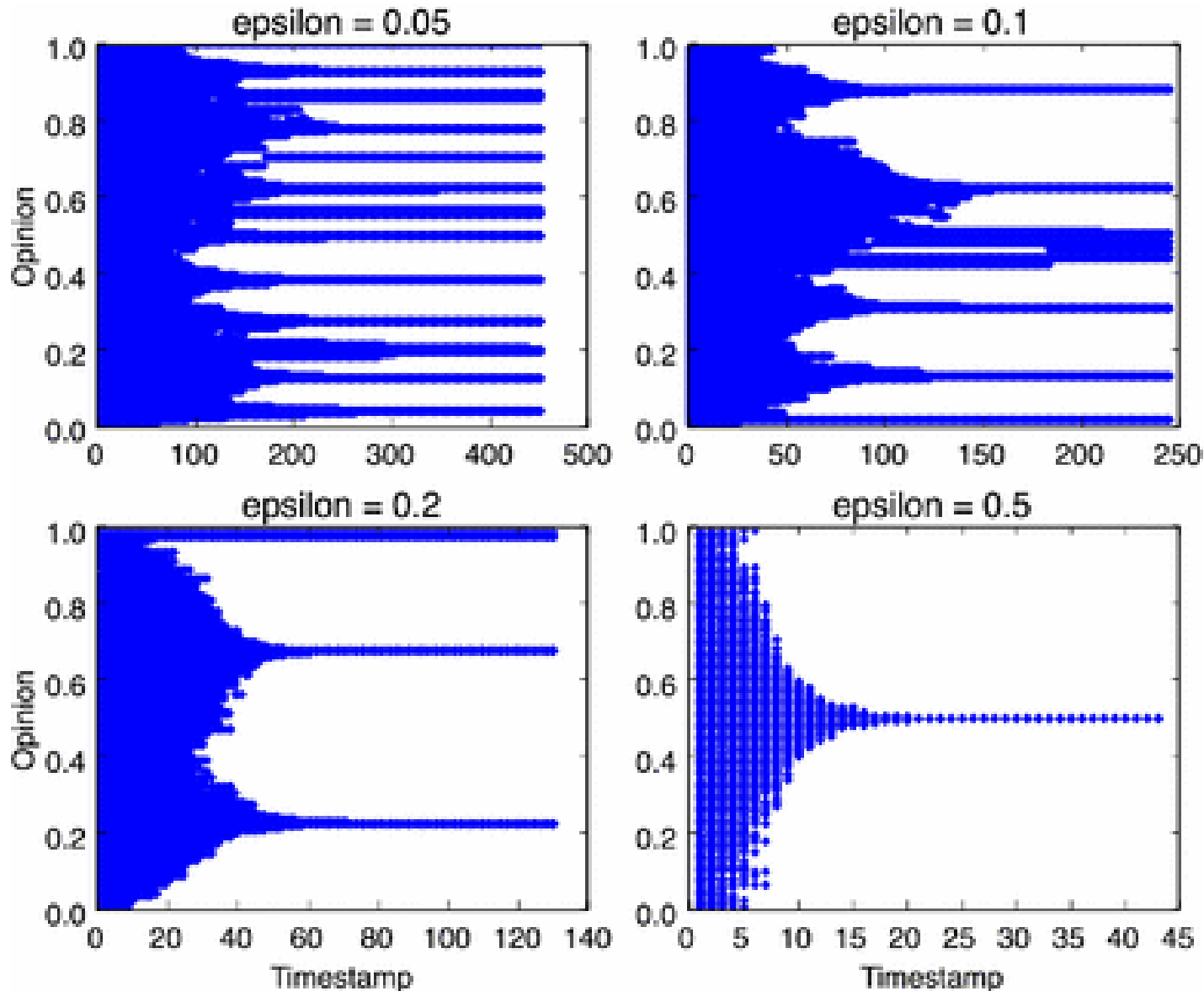


Opinion profile of a population of $N=500$ agents during its time evolution, $\epsilon = 0.25$.

The population is fully mixed, i.e., everyone may interact with everybody else.

The dynamics leads to a polarization of the population in two factions.

Behavior of the Deffuant model



Hegselmann-Krause (HK) model

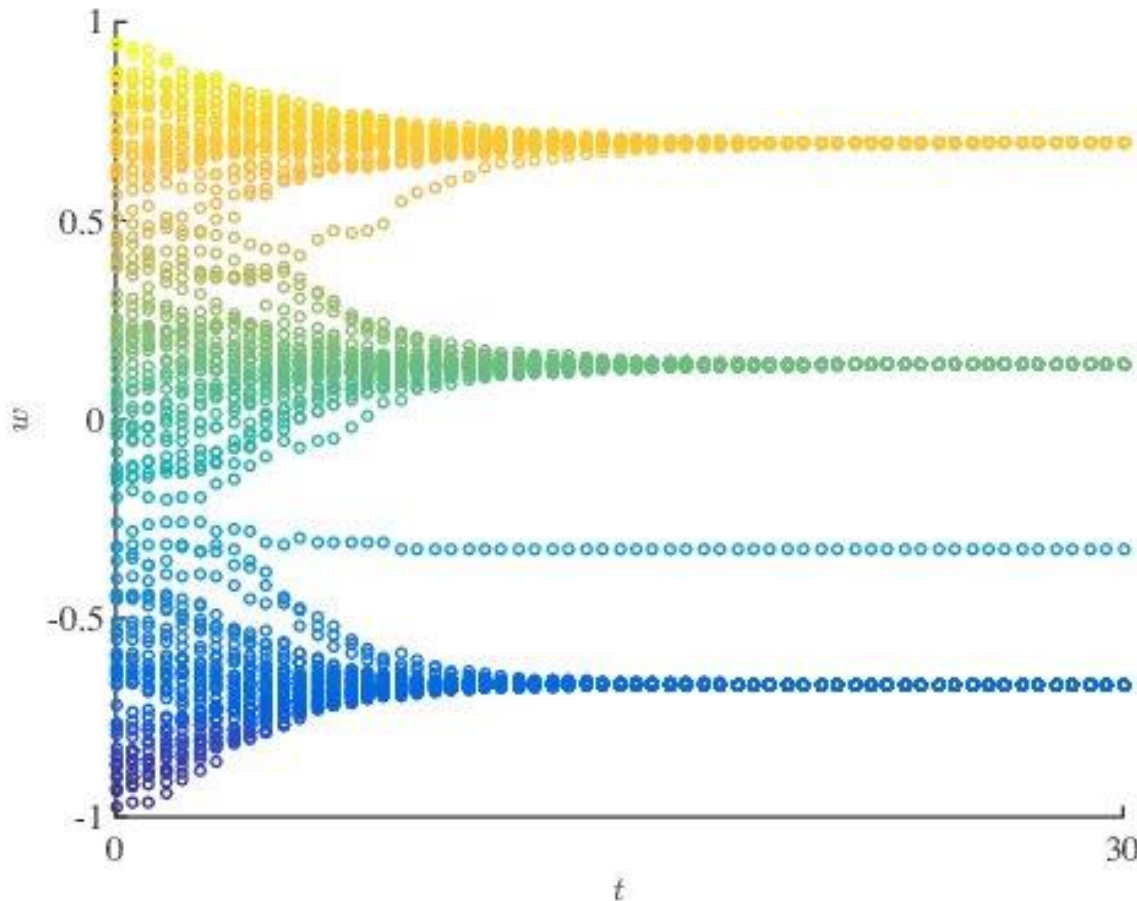
- Hegselmann and Krause, 2002
- Similarities with the Deffuant model:
 - Opinions take real values in an interval, say $[0, 1]$
 - An agent i (with opinion x_i), interacts with neighboring agents whose opinions lie in the range $[x_i - \epsilon, x_i + \epsilon]$
- **Difference:** update rule
 - An agent i does not interact with *one* of its compatible neighbors (like in Deffuant), but with *all* its compatible neighbors at once.
 - intended to describe *formal meetings*

$$x_i(t+1) = \frac{\sum_{j: |x_i(t) - x_j(t)| < \epsilon} a_{ij} x_j(t)}{\sum_{j: |x_i(t) - x_j(t)| < \epsilon} a_{ij}}$$

- a_{ij} : elements of an adjacency matrix describing the communication network.
- Agent i takes the average opinion of its compatible neighbors.

Behavior of the Hegselmann-Krause model

- fully determined by the uncertainty ε
- Need lot of computation power (due to the average calculation)



The dynamics develops similarly to the Deffuant model:

- Leads to the same pattern of stationary states, with the number of final opinion clusters decreasing if ε increases.
- for $\varepsilon > \varepsilon_c$ (a threshold) there can be only one cluster