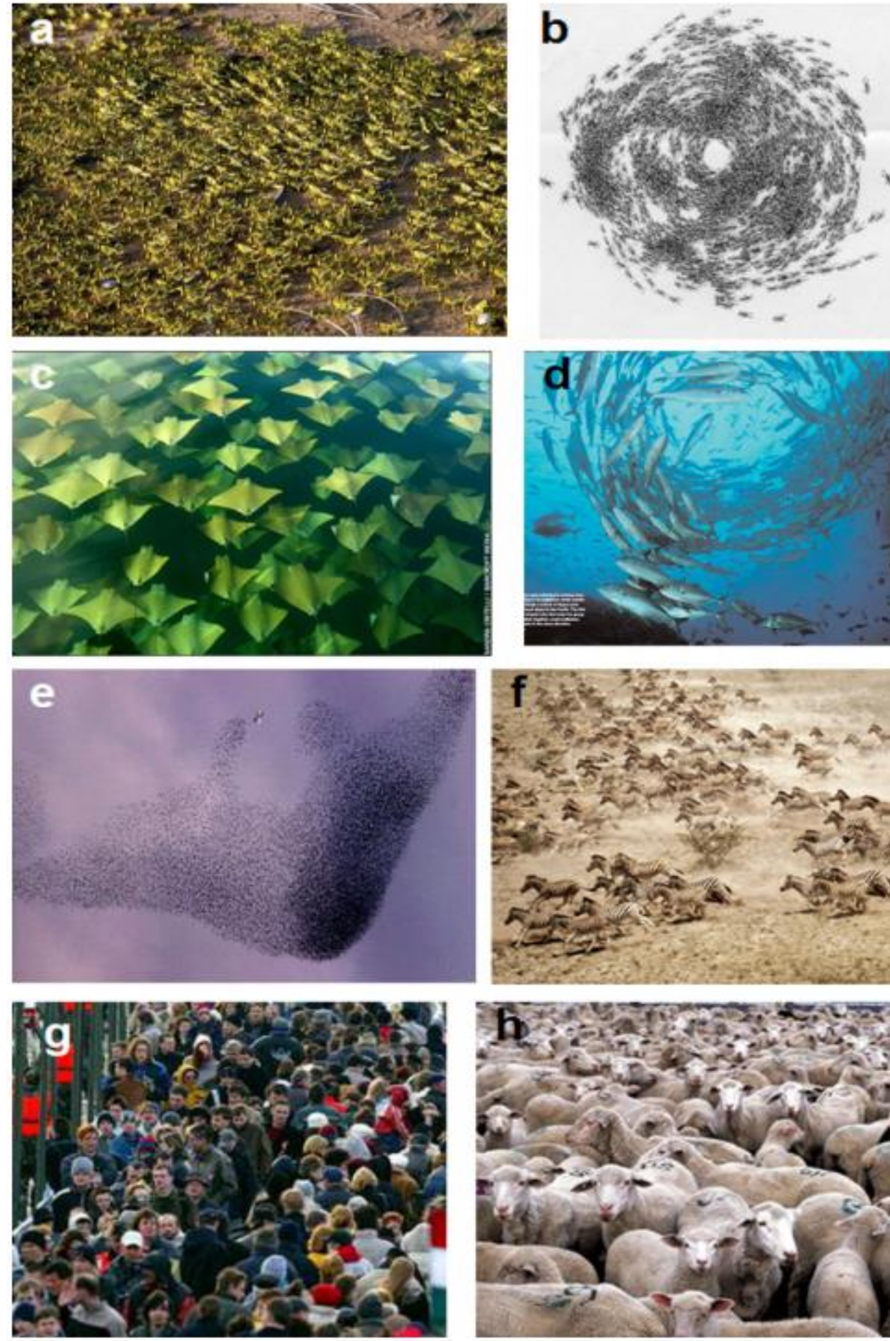
Collective motion

Sept 15, 2022

Examples

- Non-living systems (shaken rods, nano-swimmers, simple robots, boats, etc.)
- Macromolecules
- Bacteria colonies
- Cells
- Insects
- Fish schools
- Bird flocks
- Mammals
- Human crowds

- No leaders
- Spontaneous ordering
- What are the rules for self-organization?



From: T Vicsek, A Zafeiris, *Collective motion*, Physics Reports, 517, 71-140, 2012

Historical background – a new scientific field…

- One direction: computer graphics (end of 1980’s)
- Statistical physics (1990’s)*
  - Many more or less similar units
    - the concept of ***Self-propelled particles*** (SPP)
    - The assumption that the motion of the moving units are controlled by ***interactions with their neighbours***
  - Randomness (noise)
  - Spontaneous ordering

  - It was a very unconventional idea back in the 1990’s to extend the concepts of statistical physics to these active, non-equilibrium systems
  - Actuality: Lars Onsager Price 2020

* T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen and O. Shochet: Novel type of phase transition in system of self-driven particles. Physical review letters, 75(6), 1226. 1995

From a more broad point of view…

Collective motion is a form of *collective* behavior

- Strongly interdisciplinary fields
- Takes many forms. What is common:

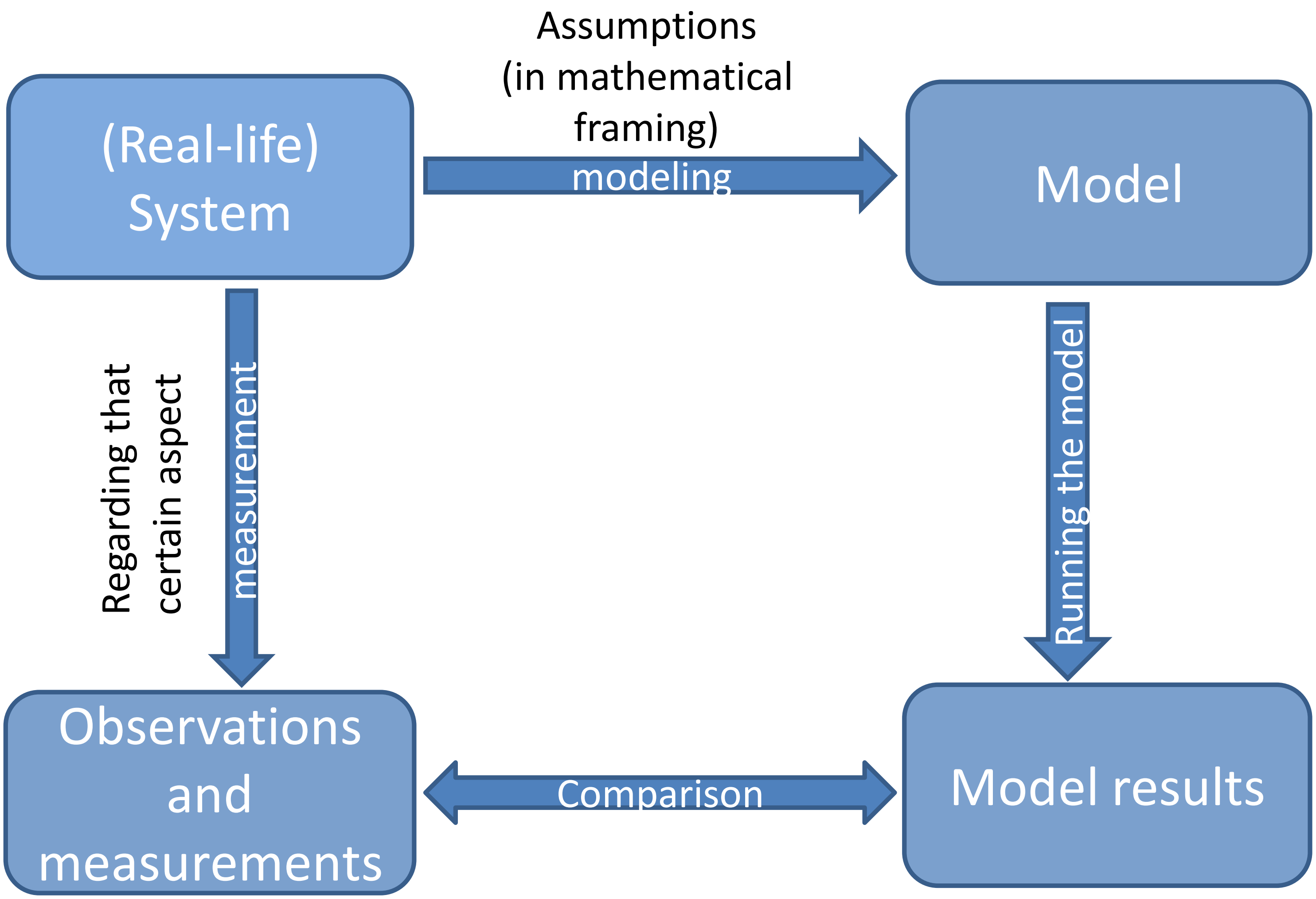
The individual behavior is strongly effected by the behavior of other group members

(→ The units behave differently in a group and alone)

Many new branches, related fields appeared

(e.g: Collective decision making)

Modeling



Assumptions
(in mathematical
framing)
modeling
(Real-life)
System
Model
Regarding that
certain aspect
measurement
Running the model
Observations
and
measurements
Comparison
Model results

Data collection techniques

- In order to yield data which is "good enough" to test model results, the *individual trajectories* of the group members have to be recorded.

- Sources of difficulties:
  - The number of units (individuals) is often high
  - They often look very much alike
  - They usually move fast

Starling video

Data collection techniques

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Starling video

Data collection techniques

- The two main factors determining the applied technology:
  1. Size of the moving units
  2. Size and direction of the space in which the group can move

  (both can range through many scales)

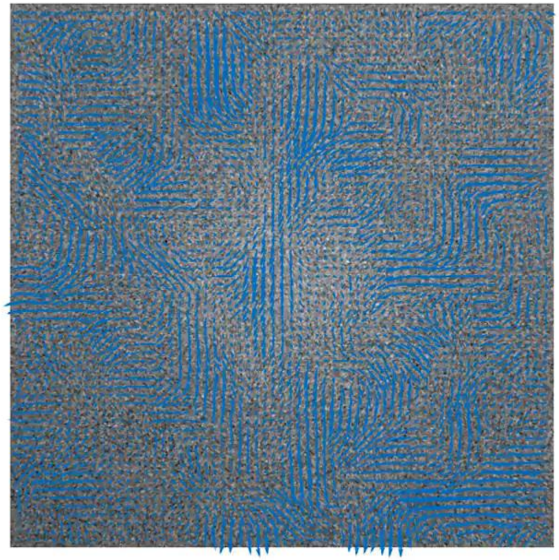
- Different techniques allow for different types of results

Data collection technique(s) - bacteria

"Particle Image Velocimetry", (PIV)

- Developed to visualize the motion of
  - small particles in
  - well-confined area

An optical technique used to produce the two dimensional instantaneous velocity vector field of fluids, by seeding the media with ***'tracer particles'***. These particles are assumed to follow the flow dynamics accurately, and it is their motion that is then used to calculate velocity information.



Bacterial Collective Motion with PIV output overlaid

https://www.youtube.com/watch?v=kCJekxCB9tM

Data collection techniques – fish

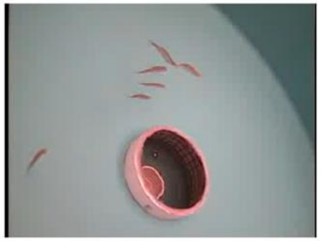
- Various size of fish
- Confined or unconfined in space

- Confined space – aquarium
  - 2D (avoiding the difficulties of 3D data)
    - container which is ‘‘basically’’ two dimensional (very shallow): 40 cm × 30 cm × 2 cm.
    - Track fish with a single video recorder.
  - 3D : three orthogonally positioned video cameras
  - ID recognition has to be solved

CDCL Tracking Fish Position and Pose II

The Collective Dynamics and Control Laboratory at the University of Maryland uses tools from projective geometry and Bayesian estimation to reconstruct the 3D position and pose of individual fish in a school.

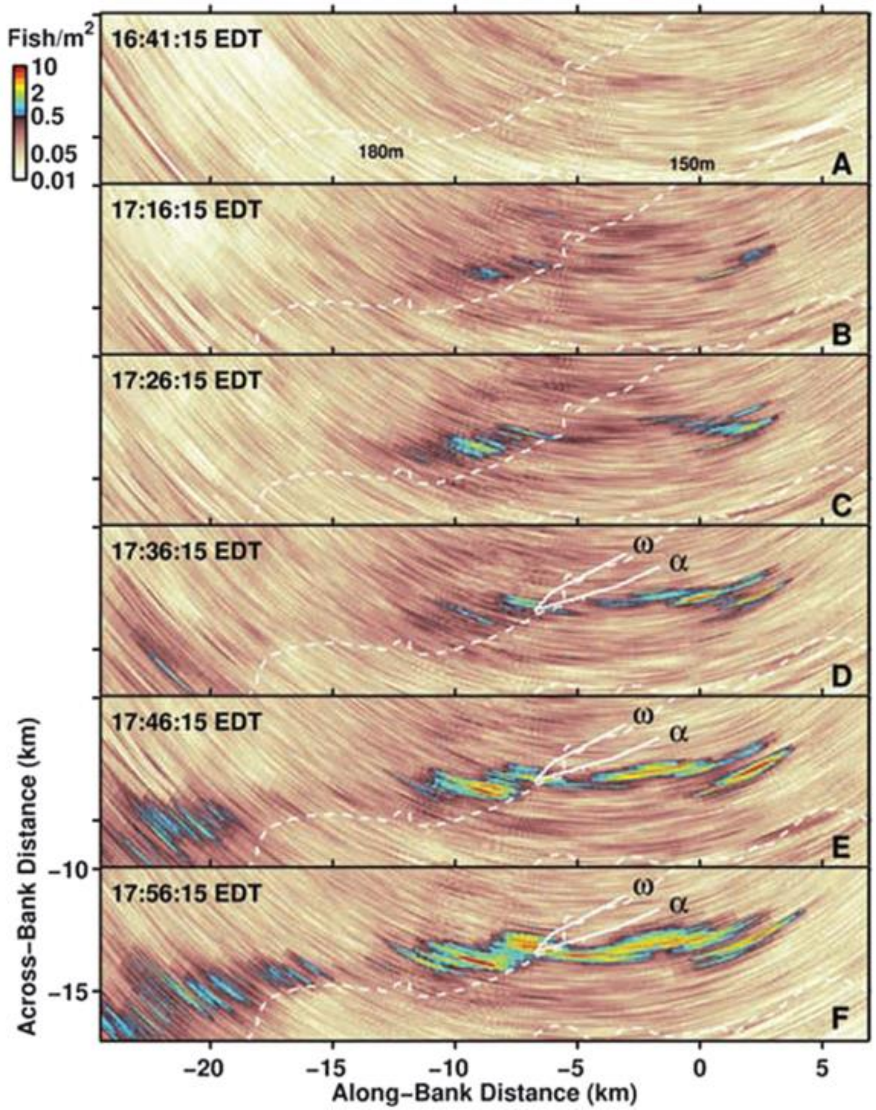
https://www.youtube.com/watch?v=QtqnMvWZfIY



Data collection techniques – fish

- Unconfined space
  - OAWRS (''Ocean Acoustic Waveguide Remote Sensing'')
  - exploits the wave propagation properties of the ocean environment
  - Instantaneous, continuous monitoring of fish populations covering thousands of square kilometers

  - (no individual recognition)
  - Results:
  - rapid transition from disordered to highly synchronized behavior at a critical density
  - small set of leaders can significantly influence the actions of a much larger group



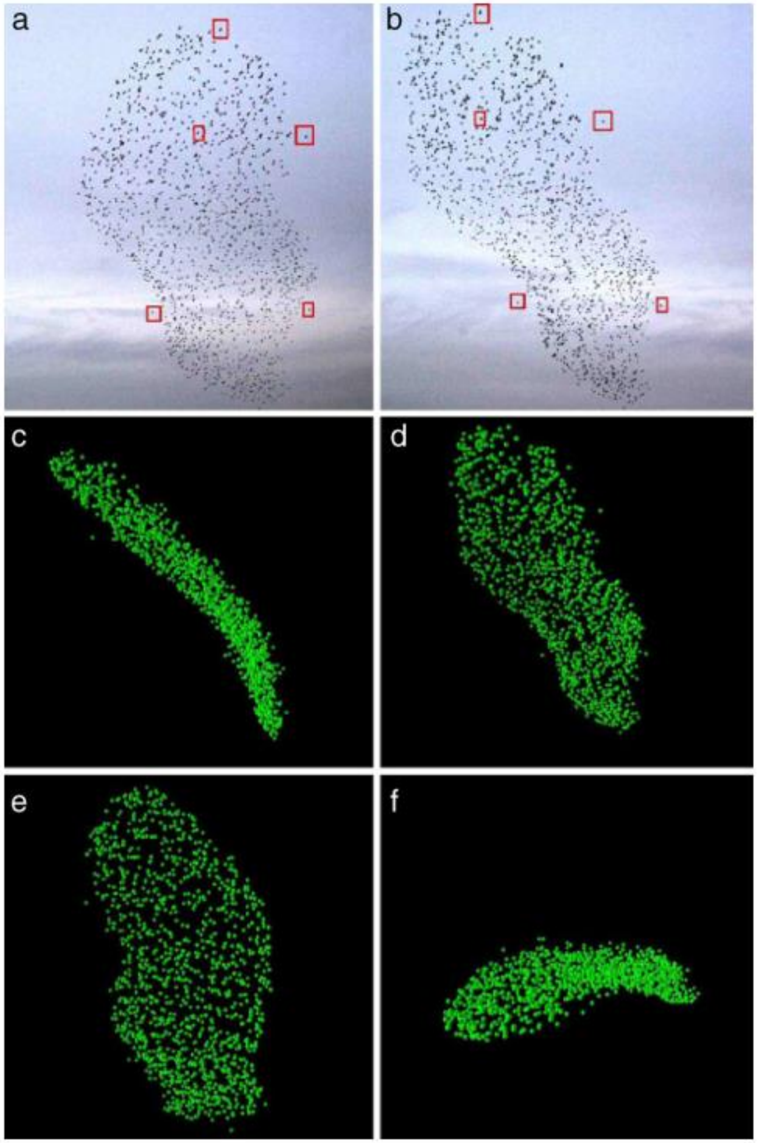
OAWRS snapshots showing the formation of vast herring shoals, consisting of millions of Atlantic herring, on the northern flank of Georges Bank (situated between the USA and Canada) on 3 October 2006. Source: Makris et al. (2009).

Data collection techniques – birds
Stereo photography technique

- Firstly: Major and Dill, 1978
- 3D positions of birds within flocks of European starlings and dunlins
- Ballerini et al. (2008) : 3D positions of up to 2,600 starlings in airborne flocks with high precision

- Pro: detailed and accurate analysis of nearest neighbor distances in large flocks
- Con: no trajectory reconstruction of the individual flock members

- Main observation: starlings in huge flocks interact with their 6–7 closest neighbors (''topological approach'') instead of those being within a given distance (''metrical approach'')



(a) and (b): stereometric photographs, taken from 25 meters apart. For reconstructing the flocks in 3D, each bird's image on the left had to be matched to its corresponding image on the right using and computer vision techniques. The small red squares indicate five of these matched pairs. (c–f) The 3D reconstructions of the analyzed flock from four different perspectives.

Source: From Ballerini et al. (2008).

Data collection techniques – birds - GPS

- Firstly: ~2006
- Record the trajectory of moving animal with high temporal resolution
- Unconfined region, natural environment

- Limits:
  - growing cost of the research with the growing number of tracked flock members
  - limited accuracy of the devices.

Data collection technique(s) – vertebrate flocks

- Bigger individuals
- Often unconfined space
- Mainly camera-based techniques
- First observations in the ’70s
  - Aerial photos – 2D
  - African buffalo herds
- Later photos were replaced with videos

- New technologies, like GPS (dogs)

- Individual recognition:
  - By hand
  - Various computer algorithms
    - Color bar technique (rats, pigeons)
      - Colors fade
      - Individuals cover each other
      - Colors depend on the actual lighting conditions

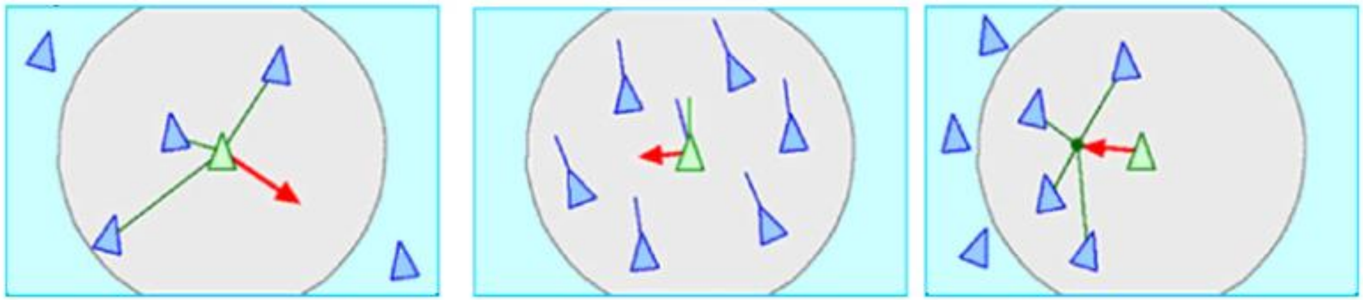
A snapshot of the processed video sequence, recording the feeding-queuing activity of a group of homing pigeons. Each bird is marked with a unique combination of three colors serving as an individual code for a computer program designed to identify the individuals automatically. Circles divide the different activity regions: central circle: feeding, blue: queuing, external circle: “not interested”. Reproduced from Nagy et al. (2013)

Basic assumptions in *collective motion* models :

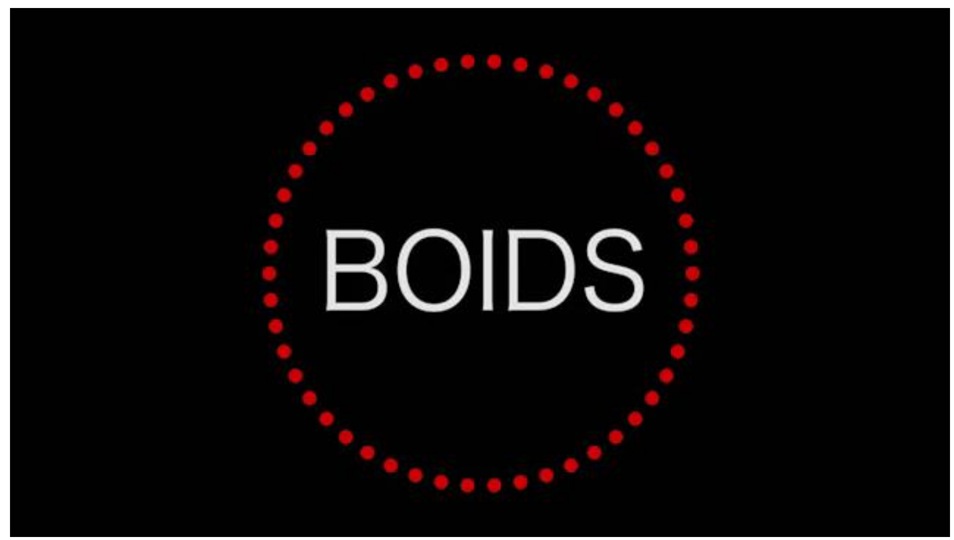
- The units are
  - Rather similar
  - moving with a nearly constant absolute velocity
  - Capable of changing their direction (including active alignment)
  - interacting within a specific range
  - subject to a noise of a varying magnitude

- SPP: "*Self-propelled* particle"
  (intrinsic source of motion)

The first models – Reynolds, 1987



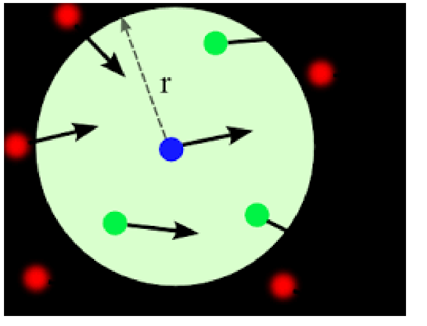
- First well-known model. (Aoki)
- Main motivation: to simulate the visual appearance of a few dozen coherently moving objects, like birds or spaceships (computational graphics)
- "boid" - "bird-like object"
- 3 types of interactions:
  - *Separation*: Avoidance of collisions
  - *Alignment*: Heading in the direction of the neighbors
  - *Cohesion*: Staying close the center of mass of the flock
- "ROI" – range of interaction



https://www.youtube.com/watch?v=QbUPfMXXQIY

SVM – “Standard Vicsek Model”

- A Statistical physics type of approach
- The units
  - move with a fixed absolute velocity v_0
  - assume the average direction of others within a given distance *R*.
- Perturbations are taken into account by adding a random angle to the average direction.
- The equations determining the motion of particle i:



$$\vec{x_i}(t+1) = \vec{x_i}(t) + \vec{v_i}(t+1)$$

$$\vec{v_i}(t+1) = v_0 \frac{\langle \vec{v_j}(t) \rangle_R}{\left| \langle \vec{v_j}(t) \rangle_R \right|} + perturbation$$

Or, in other form:

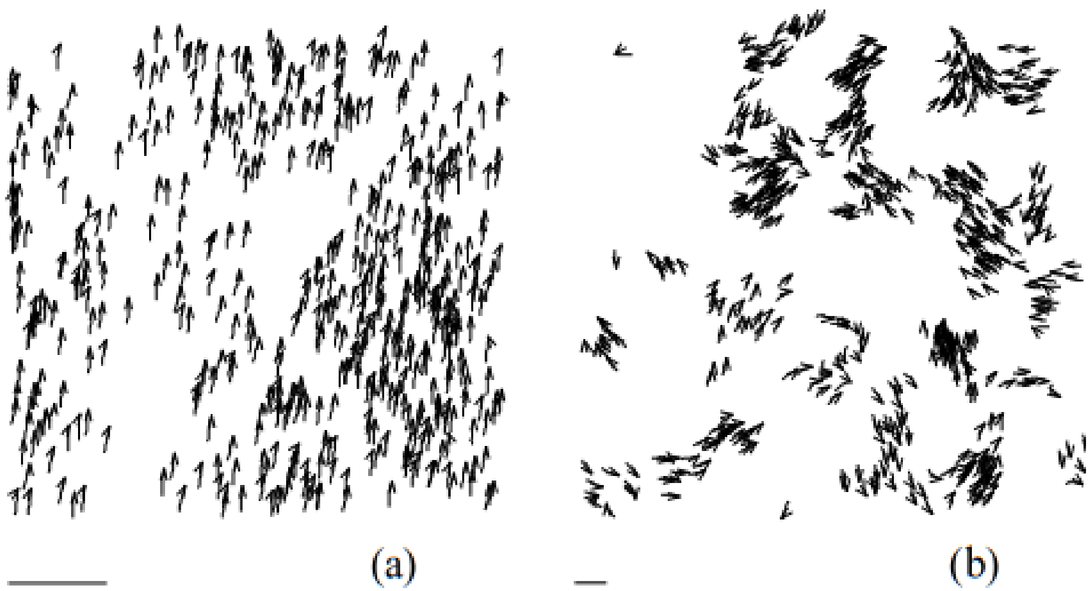
$$\vartheta_i(t+1) = \langle \vartheta(t) \rangle_{S(i)} + \xi$$

Where the noise ξ is a random variable with a uniform distribution in the interval $[-\eta/2\,, \eta/2]$.

"Novel Type of Phase Transition in a System of Self-Driven Particles". Physical Review Letters. 75 (6): 1226–1229. T. Vicsek , A. Czirók, E. Ben-Jacob, I. Cohen, O. Shochet (1995).

Parameters

- Density ρ (number of particles in a volume R^d, where d is the dimension) (or R, the interaction range)
- Velocity v_0 (fixed, same for all particles)
- Level of perturbation, η



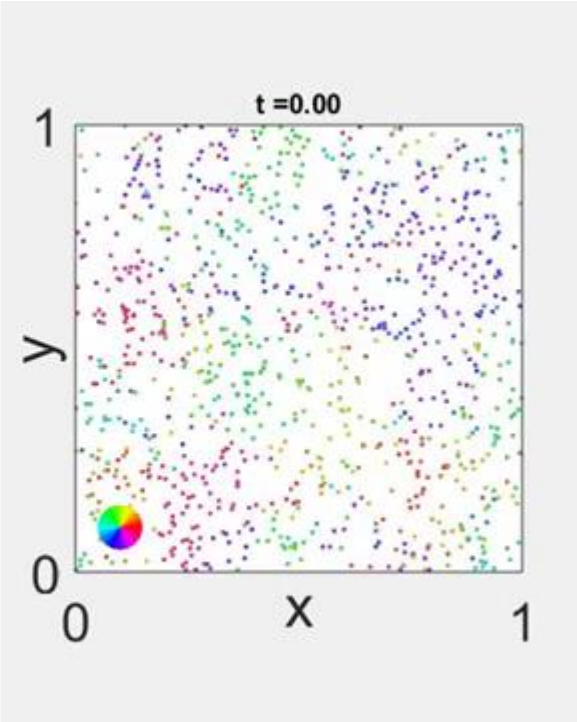
- Direction of the arrow: actual velocity
- Trajectory of the last 20 time-step: curve
- ROI: bar

Typical configurations of SPPs displayed for various values of density and noise. The actual velocity of a particle is indicated by a small arrow, while its trajectory for the last 20 time step is shown by a short continuous curve. For comparison, the radius of the interaction is displayed as a bar.

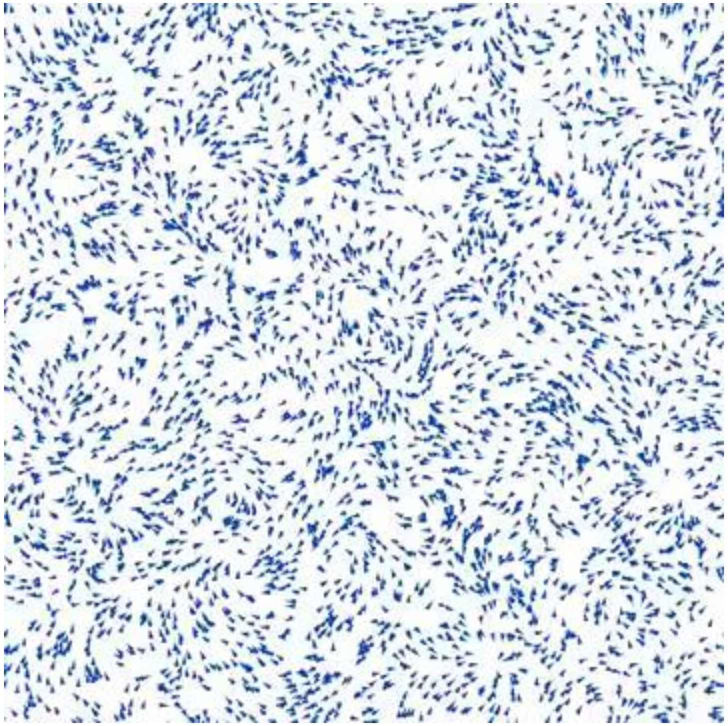
(a) At high densities and small noise (N = 300, L = 5 and η = 0.1) the motion becomes ordered.

(b) For small densities and noise (N = 300, L = 25 and η = 0.1) the particles tend to form groups moving coherently in random directions.

Parameters in the SVM

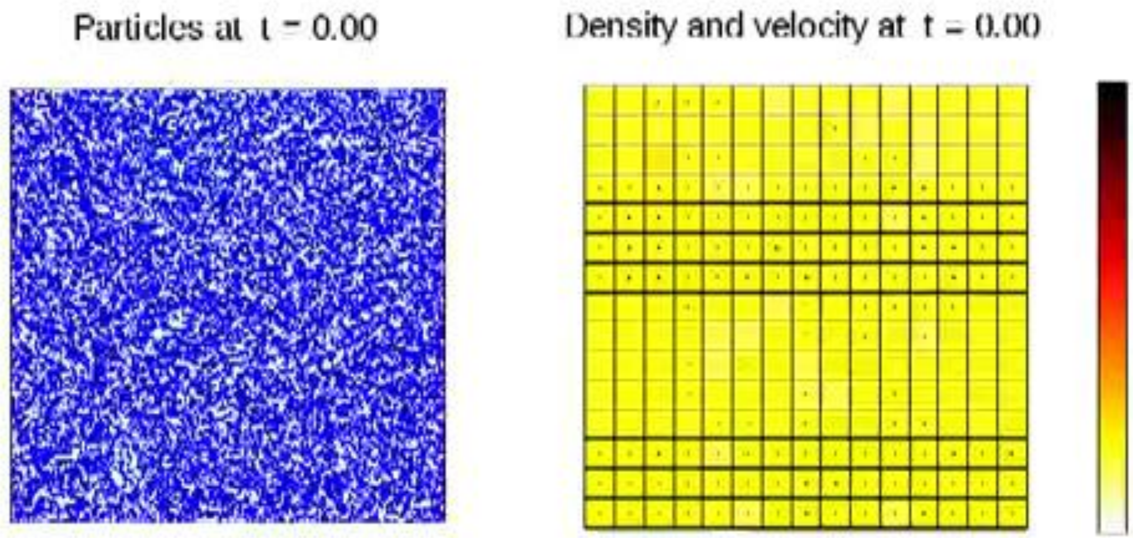


Zero noise, v_0= 0.05, R = 0.1
https://www.youtube.com/watch?v=hOI7IhjDMQ8



A system of 4.000 particles with a noise of η=0.5.
https://www.youtube.com/watch?v=Oj9L70Fh9PM

Simulation results



Left panel: particle positions and velocities. Right panel: cell-averaged particle density (color coded) and momentum density (arrows).

https://www.youtube.com/watch?v=zb0Me6GL9cM

Relation to the ferromagnetic model

Ferromagnets / SPP models

Analogies:

Ferromagnets	SPP models
Hamiltonian tending to align the spins	aligning rule (regarding the direction of motion)
Temperature	Amplitude of the random perturbation

Differences:

Ferromagnets

- Particles do not move
- There is no ordered phase in finite temperatures in 2D

SPP models

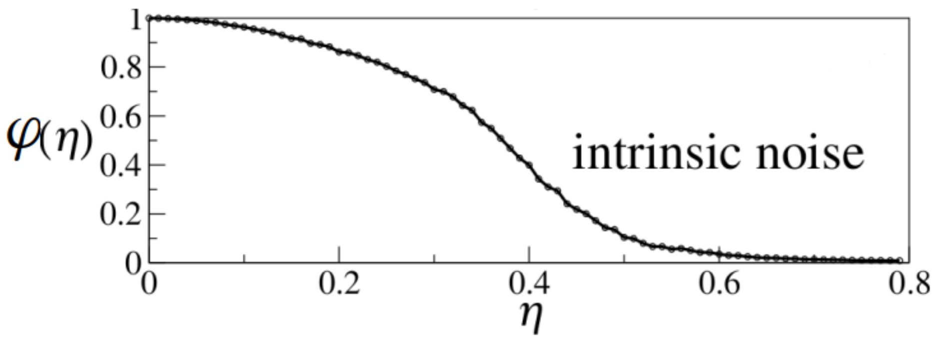
- Particles move
- Ordered phase can exist at finite noise levels in 2D SPP models

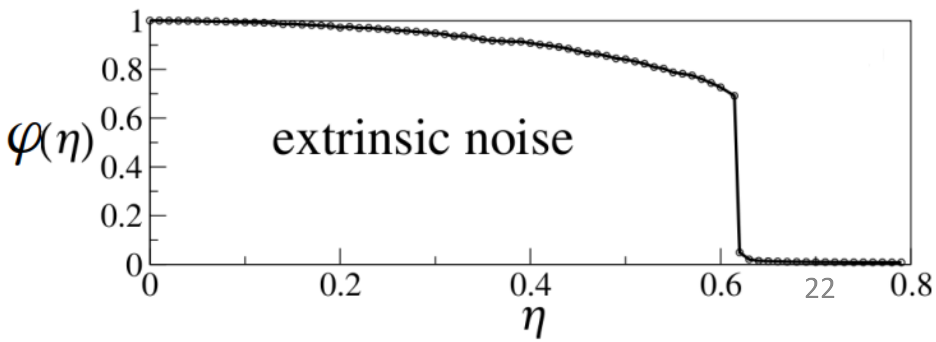
The order of the phase transition

- Order parameter: normalized average velocity

$$\varphi \equiv \frac{1}{N \cdot v_0} \left| \sum_{i=1}^{N} \vec{v_i} \right|$$

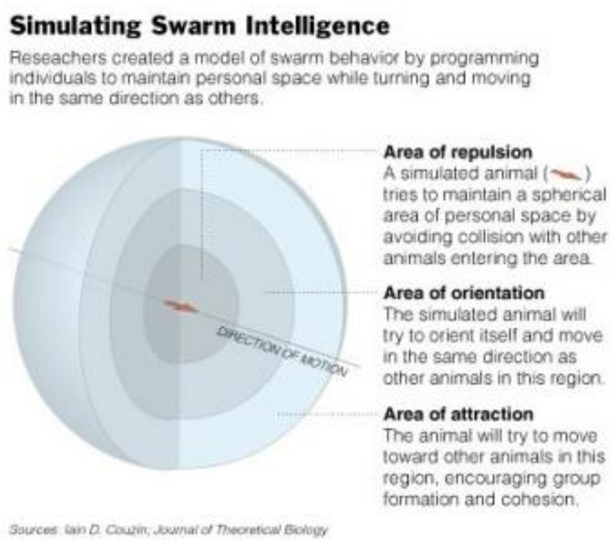
  - Non-zero in the ordered phase
  - Zero in the disordered phase
- Long debate over the nature of the transition (1st or 2nd order)
- Result: it is the magnitude of the velocity and the way the noise is introduced into the system what plays the key role
- "Intrinsic noise": the angle of the average velocity is computed and then a scalar noise is added to this angle
- "Extrinsic noise" / "Vectorial noise model": a random vector is first added to the average of the velocities and the final direction is determined only after this. When the average velocity is small, this leads to a first-order type of transition.





Moving in 3D (fish, bird) – the Couzin model

- Biologically realistic, yet still simple, individual based
- Individuals obey to the following basic rules:
  - (i) they continually attempt to maintain a certain distance among themselves and their mates,
  - (ii) if they are not performing an avoidance maneuver (described by rule *i*), then they are attracted towards their mates, and
  - (iii) they align their direction to their neighbors.
- Their perception zone (in which they interact with the others) is divided into three non-overlapping regions



- Personal space – avoiding collision

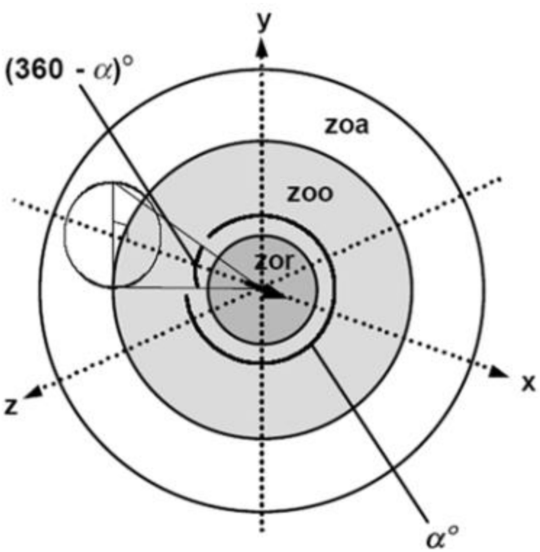
- Orientation

- Cohesion; move forward the others

Couzin ID, Krause J, James R et al, *Collective memory and spatial sorting in animal groups,* J. theor Biol, **218**, 1-11

Moving in 3D (fish, bird) – the Couzin model

The interaction zones, centered around each individual.



- ZOR, the inner-most sphere with radius R_r, is the “Zone of Repulsion”

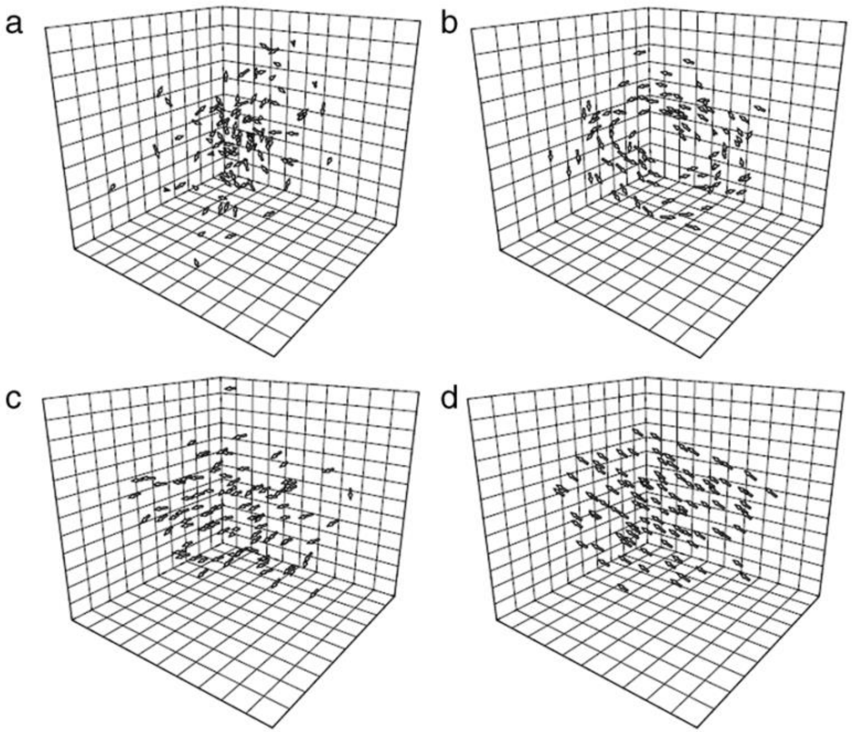
  If others enter this zone, the individual will response by moving away from them into the opposite direction, that is, it will head towards $-\sum_{j \neq i}^{n_r} \frac{\vec{r_j} - \vec{r_i}}{|\vec{r_j} - \vec{r_i}|}$, where n_r is the number of individuals being in the ZOR.

  The interpretation of this zone is to maintain a personal space and to ensure the avoidance of collisions.

- ZOO: “Zone of Orientation”. If no mates are in the ZOR, the individual aligns itself with neighbors within this ZOO region.
- ZOA: “Zone of Attraction”.

  The interpretation of this region is that group-living individuals continually attempt to join a group and to avoid being alone or in the periphery.
- α “Field of perception” (can be 360°)
- “Blind volume” behind the individual: a cone with interior angle (360-α)°. Here neighbors are undetectable.

Basic types – parameter setting

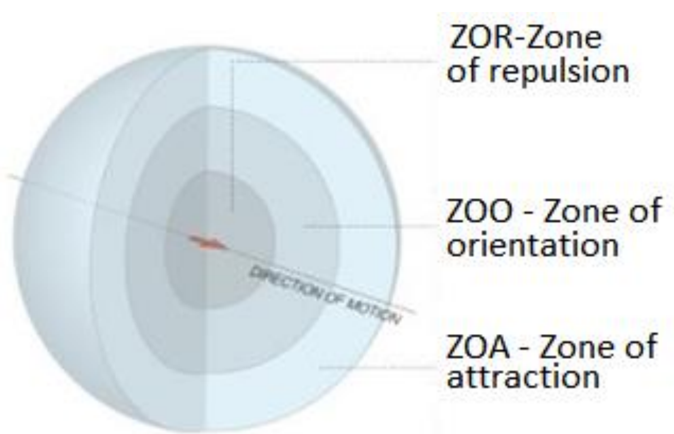


a) **Swarm**

b) **Torus** or **milling**:
big ΔR_a (width of the Zone Of Attraction)
small ΔR_o (width of the Zone Of Orientation)

c) **Dynamic parallel group**:
intermediate to high ΔR_a (width of the Zone Of Attraction)
intermediate ΔR_o (width of the Zone Of Orientation)

d) **Highly parallel group**:
increasing ΔR_o further (width of the Zone Of Orientation)



ΔR_a: width of the ZOA
ΔR_o: width of the ZOO

Couzin model – cont.

- System properties:
  - Order parameter:

$$\varphi(t) = \frac{1}{N}\left|\sum_{i=1}^{N} \overrightarrow{v_i{}^{u}}(t)\right|$$

  - (group) anguar momentum:

$$m_{Gr}(t) = \frac{1}{N}\left|\sum_{i=1}^{N} \vec{r}_{i-Gr}(t) \times \overrightarrow{v_i{}^{u}}(t)\right|$$

(average of the angular momenta of the group members around the center)

- $\overrightarrow{v_i{}^{u}}$ is *unit* direction vector of individual i, so
- φ (order param) is the same as in the SVM

- $\overrightarrow{r_{Gr}}$ position of the group center
- $\vec{r}_{i-Gr} = \vec{r}_i - \vec{r}_{Gr}$ vectorial difference of the position of individual i and the group center
- Group center:

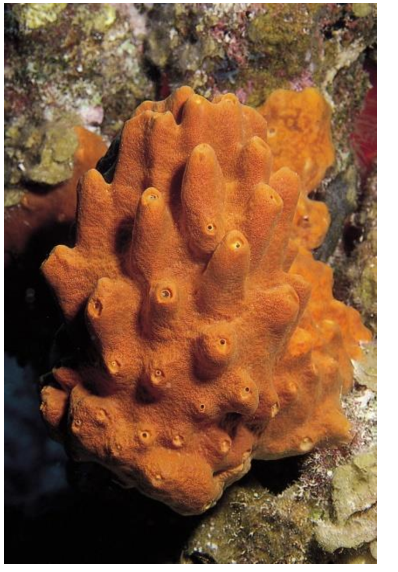
$$\vec{r}_{Gr}(t) = \frac{1}{N}\sum_{i=1}^{N} \overrightarrow{r_i}(t)$$

General (minimal) vs. system specific models

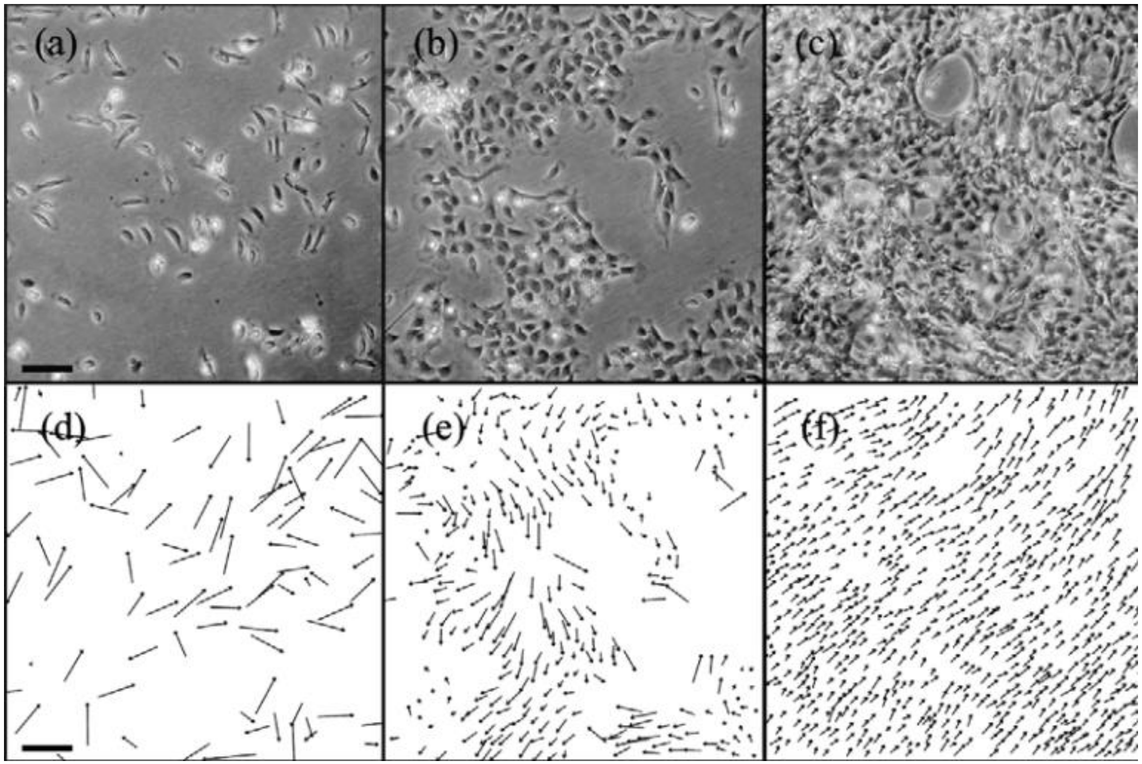
- General models: Few parameters, few assumptions ("minimal"), general results
- System specific models: include system-specific details
  - Individuals with different properties (segregating units)
  - Insect migration (e.g. locusts)
  - Predator-prey systems
  - Etc.
- Applications (among many):
  - Robotics / military applications
  - Traffic simulation
    - Vehicular traffic
    - Pedestrian motion (urban design, building design)
    - Panic

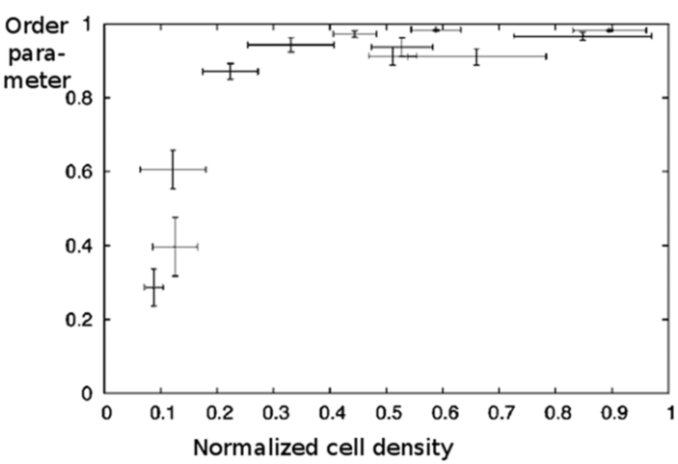
The role of adhesion

- Mechanism determining tissue movements?
  - Dates back to the beginning of the 20th century
  - 1907 Wilson discovered that sponge cells which have been previously squeezed through a mesh of fine bolting-cloth reunite again reconstituting themselves into a functioning sponge
  - Early studies
    - Cell sorting is a resultant of inhomogeneities in the immediate environment (for example of pressure)
  - Since then
    - the movements are due to intrinsic properties of the individual tissues themselves



Collective behavior of fish keratocytes





Observations / measurements

The collective behavior of fish keratocytes for three different densities. The normalized density, $\bar{\rho}$ is defined as $\bar{\rho} = \rho/\rho_{max}$, where ρ_{max} is the maximal observed density, 25 cells/100 × 100 μm2.

(a) ρ = 1.8 cells/100 × 100 μm2 corresponding to $\bar{\rho}$ = 0.072

(b) ρ = 5.3 cells/100 × 100μm2 which is $\bar{\rho}$ = 0.212, and

(c) ρ = 14.7 cells/100 × 100 μm2, $\bar{\rho}$ = 0.588. The scale bar indicates 200 μm. As cell density increases cell motility undergoes to collective ordering. The speed of coherently moving cells is smaller than that of solitary cells. (d)–(f) on the bottom panel depicts the corresponding velocities of the cells.

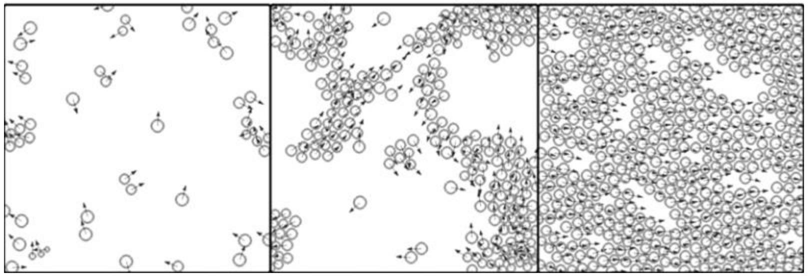
Order parameter versus the normalized cell density. The error bars show the standard error of the density and order parameter.

Source: Szabó, B., Szőlősi, G.J., Gönci, B., Jurányi, Zs., Selmeczi, D., Vicsek, T., 2006. Phase transition in the collective migration of tissue cells: experiment and models. Physical Review E 74, 061908.

Collective behavior of fish keratocytes – The model

- The model cells are self-propelled particles (SPP)
- Short-range attractive–repulsive inter-cellular forces account for the organization of the motile keratocyte cells into coherent groups.
- Direction of the cells: according to the net-force acting on them.
- 2D flocking model:
  - *N* SPPs move with a constant speed v_0 and
  - mobility μ
  - in the direction of the unit vector $\boldsymbol{n}_i(t)$ (can be described by $\vartheta_i^n(t)$ as well)
  - while the *i* and *j* particles experiences the inter-cellular force $\mathbf{F}(\mathbf{r}_i\mathbf{r}_j)$.
  - The motion of cell $i(\in 1, \ldots, N)$ in the position $\boldsymbol{r}_i(t)$ is described by

$$\frac{d\vec{r}_i(t)}{dt} = v_0\vec{n}_i(t) + \mu\sum_{j=1}^{N}\vec{F}(\vec{r}_i\vec{r}_j)$$



Simulation results obtained by solving the equations with periodic boundary conditions.
The model exhibits a continuous phase transition from disordered to ordered phase.

Many authors put much emphasis on the actual *shape* and *plasticity* of the cells as well

Source: Szabó, B., Szőllősi, G.J., Gönci, B., Jurányi, Zs., Selmeczi, D., Vicsek, T., 2006. Phase transition in the collective migration of tissue cells: experiment and models. Physical Review E 74, 061908.

Models with segregating units

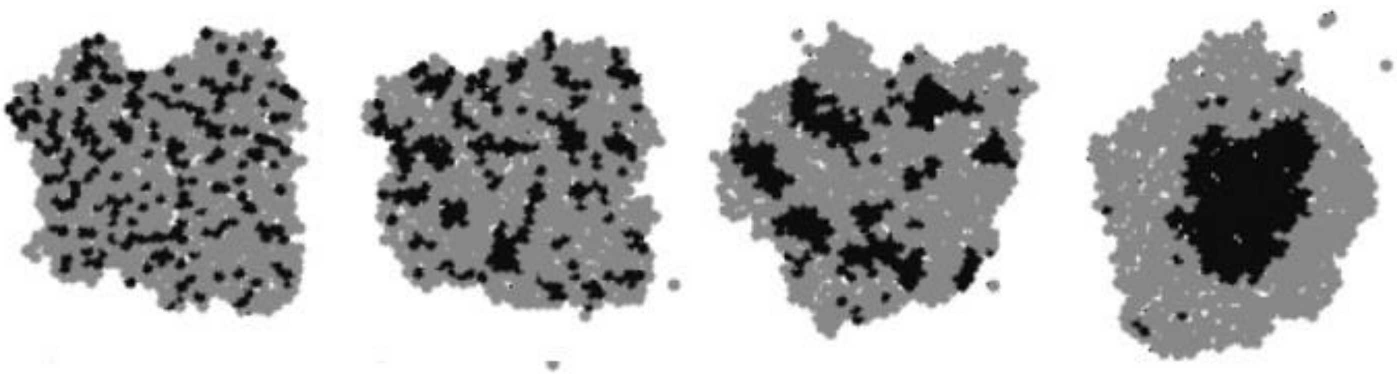
- Special case: an originally heterogeneous mixture of units segregate into two (or more) homogeneous clusters without any kind of external field.

- 2D example:
  - Granular segregation
  - Cell sorting
    - development of organs in an embryo
    - regeneration after tissue dissociation

Granular segregation in a shallow container

Granular segregation in shallow container
(Nicolás Rivas)
Perfect hard spheres inside a shallow, quasi-two-dimensional container, vibrated in the vertical direction. Two types of particles: blue ten times heavier than red ones (same size). Periodic boundary conditions.
Paper: http://iopscience.iop.org/1367-2630/1...

https://www.youtube.com/watch?v=l0Aea1EWcCI&t=5s



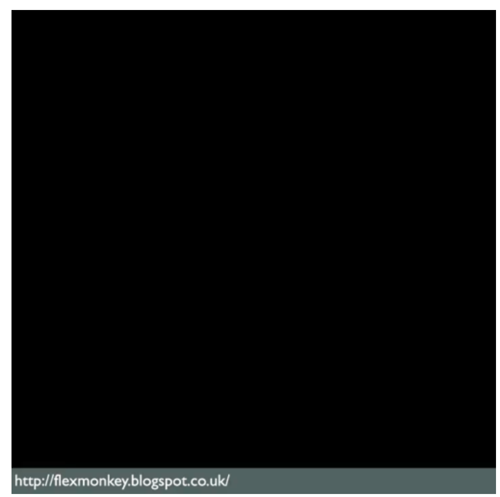
- two kinds of cells, differing in their interaction intensities.
- 800 cells

Models with segregating units

- diverse particles (behavioral / motivational) exhibit sorting:
  - relative positional change, according to the actual inner state
  - *Relative* differences play a key role
  - If the individual variations are persistent then the group will reassemble to its' original state after perturbations
- "Swarm chemistry" – by Hiroki Sayama

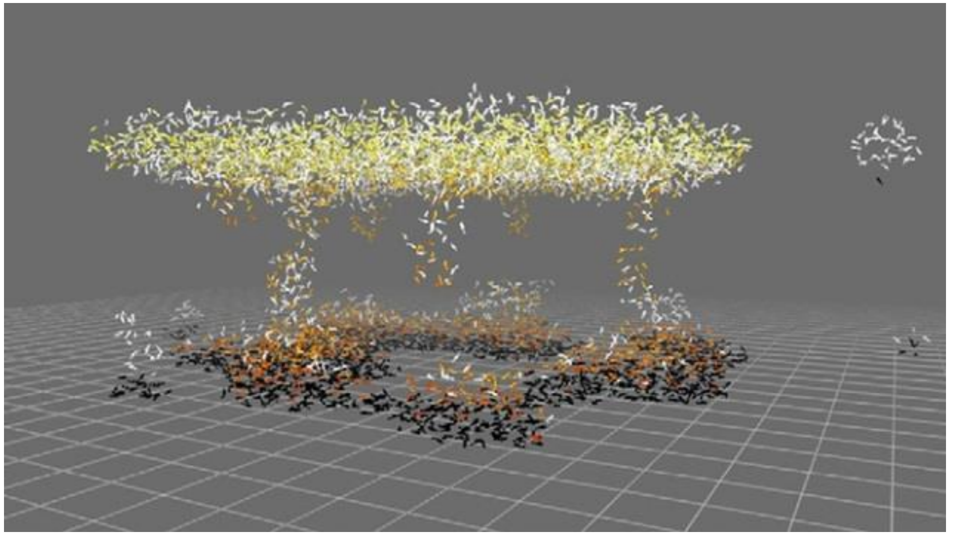
Homepage: http://bingweb.binghamton.edu/~sayama/SwarmChemistry/

- Emergent patterns in systems of particles with different kinetic parameters
  - Preferred speed, ROI, etc.
  - Infinite 2D space



Model: school of spawning herrings (Vabo & Skaret 2008)

- 3D individual-based model
- Units differed in their motivational level
  (controlled by a parameter)
- The motion of each individual:
  - (1) avoiding boundaries
  - (2) social attraction
  - (3) social repulsion
  - (4) moving towards the bottom to spawn
  - (5) avoiding predation



Results:

- Similar motivational levels results an integrated school, diverse inner states produce a system with frequent split-offs.
- Intermediate degree of homogeneity: More complex structures, like layers connected with vertical cylindrical shaped schools
  - describing the observations (Axelsen et al., 2000) allowing ovulating herring to move across the layers

→ the level of motivational synchronization among fish determines the unity of the school

Case study: Pedestrian motion; Models and their relations

- Always 2D (↔vehicle 1D)

- Traffic models are usually categorized according to the scale of the variables of the model:
  - Macroscopic,
  - Microscopic, and
  - Mesoscopic

Fredrik Johansson, Microscopic Modeling and Simulation of Pedestrian Traffic, Department of Science and Technology, Linköping University, 2013

Macroscopic models / continuum dynamic approach

- Describes the macroscopic (or average) properties of the system
- Assumes that traffic can be regarded as a *fluid*, or continuum, disregarding the fact that it is composed of discrete entities such as cars or pedestrians
  - No explicit reference to the underlying microscopic nature, → no personal preferences
  - Central assumption:
    - no (sufficiently little) significant information is lost when the microscopic details are averaged out
    - the units are identical, unthinking elements
  - successful approach in physics
  - Bit less well founded in traffic modeling, but has been successful, primarily in car traffic modeling
- The basis of fluid dynamic models of pedestrian traffic is the two dimensional continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{q} = 0$$

where ρ : mean density ($\rho = \rho(\boldsymbol{r}, t)$),

$\boldsymbol{q} = \rho u$: mean flow ($\boldsymbol{q} = \boldsymbol{q}(\boldsymbol{r}, t)$),

u : mean speed (the assumption that *u* is a function of the density, comes from observations)

Mesoscopic models

- Each individual is represented individually and can have individual properties (↔ Macroscopic)
- But the individual walker’s behavior is still determined by average quantities

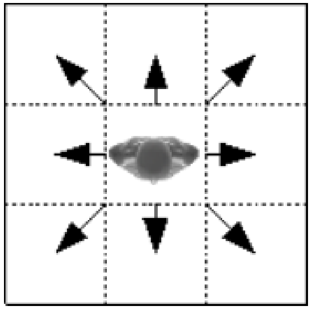
Microscopic models

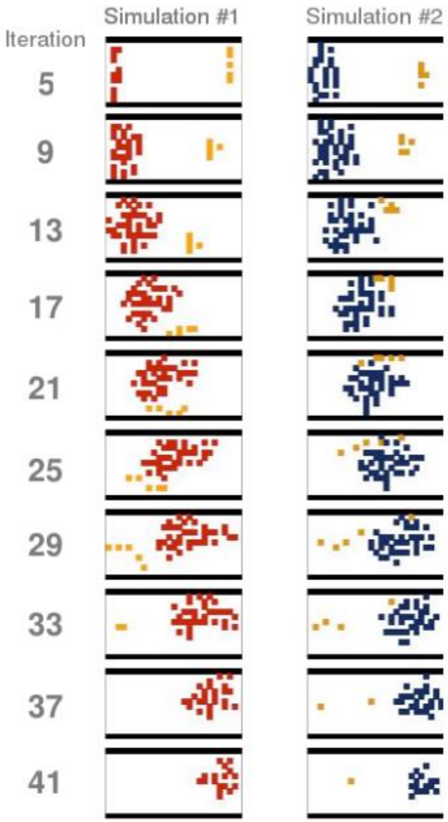
- describe every individual walker and its interaction with other walkers and the environment
- there is no averaging process → the heterogeneity of the population can be explicitly included (personal drives, motivations, preferred directions, etc.)

- Four basic types (partially overlapping, not well defined)
  1. cellular automaton based models
  2. agent based models
  3. game theoretic models
  4. force based models (Social force model)

(1) Cellular automation based models

- Very first models (1980’s), but still in use
- Discrete in space and time
- Each unit is a cell, either occupied by a pedestrian (or obstacle) or empty
- At each time step, pedestrians move into one of the neighboring cells or stay where they are.
- Limitation:
  - the size of a walker is fixed and constant over the population
  - Discrete size of movement at a time
    (but different speeds and goals can be considered)
- Pro-s:
  - Computational efficiency
  - Simple update rules → some general are easy to obtain
  - The grids can be refined
- One of the earliest models: Gipps and Marksjö (1985): (the “basics”)
  - grid with quadratic cells
  - The preferred next cell is the one that reduces the remaining distance to the walker’s destination the most
  - The navigation is modified by the presence of other walkers: repulsive potential around each walker





(2) Agent based models

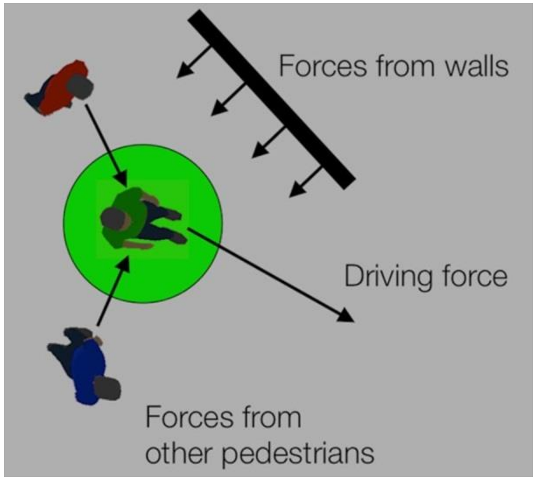
- basically CA models with "very complex" update rules
    - can be either continuous or discrete, both in space and time
    - can be governed by practically any type of behavioral rules.
    - often have a large set of behavioral rules, each dedicated to a specific situation.
    - The update procedure occurs in two steps:
        1. the agent determines the situation it is in by one or several test
        2. Executes the rule connected to that situation
    - Pro: can be very detailed
    - Con: high computational cost, hard to analytically provide properties

(3) Game theoretic models

- Movement is an “action”
- Each pedestrian plans his/her path according to her beliefs about how other pedestrians will move in the future.
  - Example:
    - Pre defined strategies
    - an empirical distribution over the strategies of other players
    - Etc.

(4) Force based models/social force models (SFM)

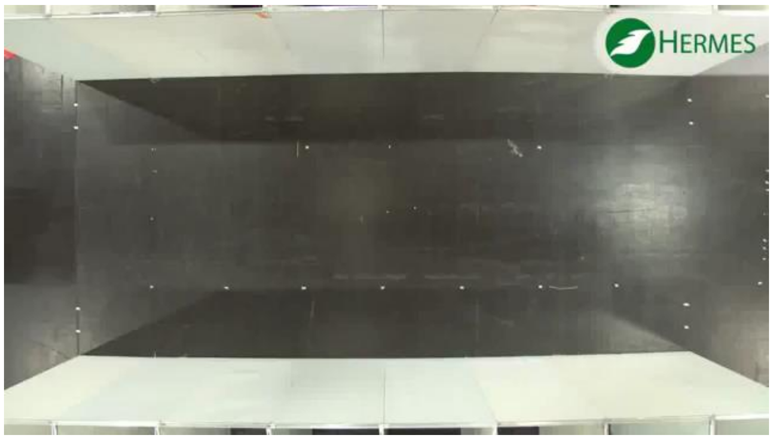
- Helbing and Molnár (1995)
- People walk in crowded environments by using automatic (subconscious) strategies for avoiding collisions and keeping comfortable distances
- These automatic strategies can be encoded as simple behavioral rules

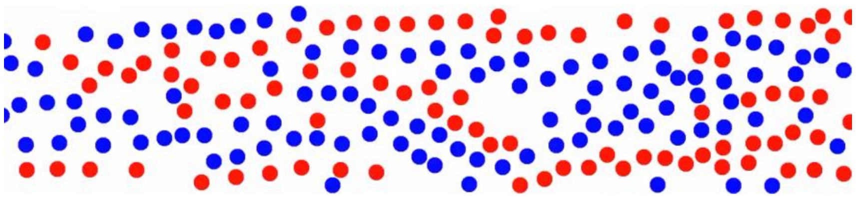


Main idea: the influences of elements of the environment on the behavior of the pedestrians appear as social forces.

- Social forces are not “real” forces (in a Newtonian meaning), rather, are a description of the motivation of the pedestrian to change its velocity, induced by some elements in the environment.
- the effects of several social forces, just like regular forces, are assumed to add as vectors
- Operates in continuous space, allowing detailed representation of the geometry of the environment
- proven to reproduce several well known features of pedestrian traffic:
  - dynamic lane formation in opposing flows
  - oscillations at bottlenecks
  - evacuation scenarios

Dynamic lane formation in opposing flows





Experiment:

Walkers self-organize into lanes to avoid interactions with oncoming pedestrians. This helps them to move faster than is otherwise possible. This happens effortlessly and requires no communication
https://www.youtube.com/watch?v=J4J__lOOV2E

Model:

F. Zanlungo, T. Ikeda and T. Kanda,
Social force model with explicit collision prediction,
Europhysics Letters, Volume 93, 68005

https://www.youtube.com/watch?v=u2kEM2Ed6Xk

An application for SFM: Panic in human crowd

According to the socio-psychological literature the characteristic features of escape panics:

(1) People try to move considerably faster than normal
(2) Individuals start pushing, and interactions become physical.
(3) Moving and passing of a bottleneck becomes uncoordinated.
(4) At exits arching and clogging are observed.
(5) Jams build up
(6) The physical interactions add up and cause dangerous pressures up to 4,450 $^{N}/_{m^2}$ which can bend steel barriers or push down brick walls

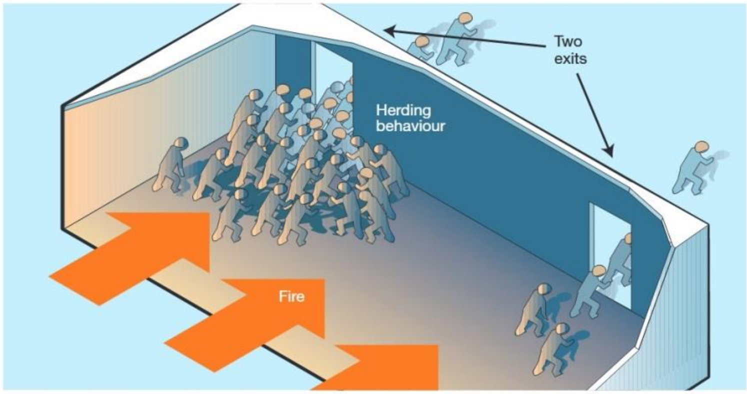
Dirk Helbing, Illés Farkas, and Tamás Vicsek: *Simulating dynamical features of escape panic*. Nature 407, 487-490 (2000)

Model: Panic in human crowd

- Many-particle SPP system
- Main assumption: the individual behavior is influenced by a mixture of socio-psychological and physical forces

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0(t)\mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}$$

N: number of pedestrians (size of the crowd)
m_i: mass of the i-th pedestrian
${v_i}^0$: desired speed of individual i
${\boldsymbol{e}_i}^0$: preferred direction of individual i
$\boldsymbol{v}_i(t)$: actual velocity
τ_i: characteristic („reaction") time of individual i
f_{ij} and f_{iW}: „interaction forces": individual i tries to keep a velocity-dependent distance from other pedestrians j and walls W.



How crowd behaviour affects escape from a smoke-filled room. Previous simulations of pedestrian behaviour in crowds have used a model based on fluid flow through pipes, but these ignored the actions of individuals. According to the individual-centred model of Helbing *et al.*[1], the evacuation of pedestrians from a smoke-filled room with two exits can lead to herding behaviour and clogging at one of the exits. By contrast, a traditional fluid-flow model would predict the efficient use of both exits. A more individual-centred approach is required to reproduce the behaviour of real crowds.

Panic model – cont.

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0(t)\mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}$$

The psychological tendency of pedestrians i and j to avoid each other: repulsive interaction force:

$$A_i e^{\frac{r_{ij}-d_{ij}}{B_i}} \boldsymbol{n}_{ij}$$

If $d_{ij} < r_{ij}$ then the pedestrians touch each other. In this case two additional forces (after granular interactions):

1. "Body force":

   $k(r_{ij} - d_{ij})\boldsymbol{n}_{ij}$

   counteracting body compression
2. "Sliding friction force"

   $\kappa(r_{ij} - d_{ij})\Delta v_{ij}{}^t \boldsymbol{t}_{ij}$

   impeding relative tangential motion

   $\boldsymbol{t}_{ij}$ is the tangential direction, and

   $\Delta v_{ij}{}^t = (\boldsymbol{v}_j - \boldsymbol{v}_i) \cdot \boldsymbol{t}_{ij}$ is the tangential velocity difference

N: number of pedestrians (size of the crowd)
m_i: mass of the i-th pedestrian
$v_i{}^0$: desired speed of individual i
$\boldsymbol{e}_i{}^0$: preferred direction of individual i
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f_{ij} and f_{iW}: „interaction forces": individual i tries to keep a velocity-dependent distance from other pedestrians j and walls W.

$\boldsymbol{r}_i(t)$ position of individual i
A_i constant
B_i constant
$d_{ij} = \|\boldsymbol{r}_i - \boldsymbol{r}_j\|$ distance between the pedestrians' center of mass
$\boldsymbol{n}_{ij}$: normalized vector pointing from pedestrian j to i
r_i : the radius of pedestrian i
$r_{ij} = r_i + r_j$ the sum of the radii of pedestrians i and j

κ : constant (large)
k : constant (large)

$g(x)$: zero, if the pedestrians do not touch each other ($d_{ij} > r_{ij}$), Otherwise equal to the argument x.

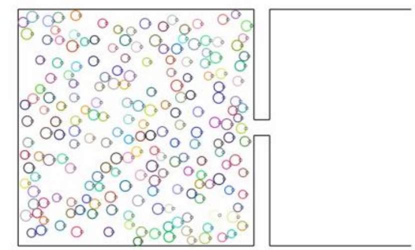
$$f_{ij} = \left\{A_i e^{\frac{r_{ij}-d_{ij}}{B_i}} + k \cdot g(r_{ij} - d_{ij})\right\}\boldsymbol{n}_{ij} + \kappa g(r_{ij} - d_{ij})\Delta v_{ij}{}^t \boldsymbol{t}_{ij}$$

Simulation results with reasonable parameters

1. Transition to incoordination due to clogging.

   The outflow from a room is well coordinated and regular desired velocities are normal.

   But for desired velocities above 1.5 $^{m}/_{s}$ (rush) an irregular succession of arch-like blockings of the exit and avalanche-like bunches of leaving pedestrians when the arches break appear.

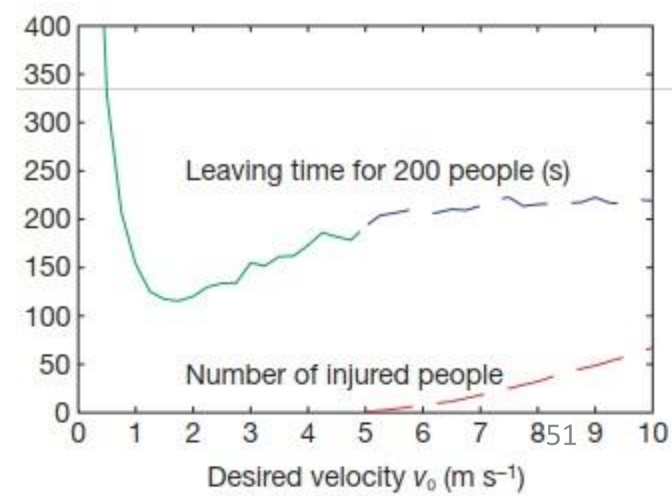


Simulation of 200 pedestrians evacuating a 15x15m room passing through a 1meter-wide door at a desired speed of 3.5m/s.

https://www.youtube.com/watch?v=FidqTZiJvRA

2. "Faster-is-slower" effect due to impatience. Since clogging is connected with delays, trying to move faster can cause a smaller average speed of leaving (κ is large)

   \- fire



Simulation results with reasonable parameters

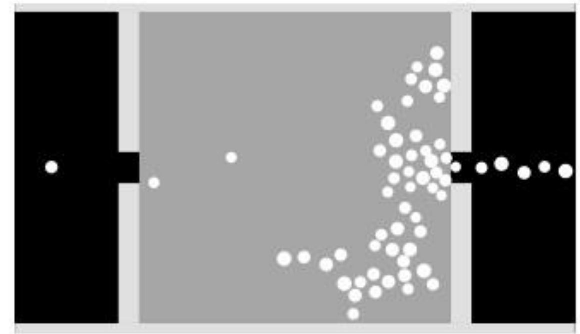
3. **Mass behavior.** Simulated situation: pedestrians are trying to leave a smoky room, but first have to find one of the invisible exits.

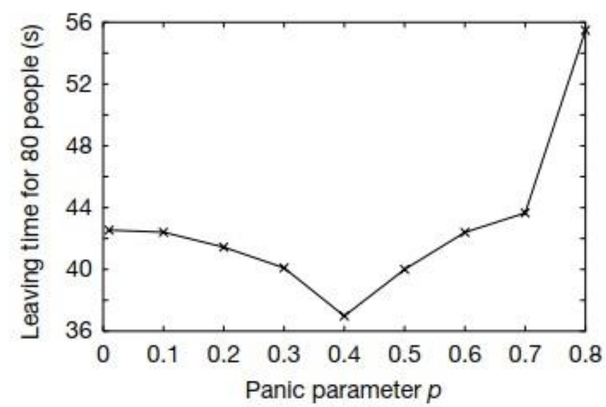
Each pedestrian i may either

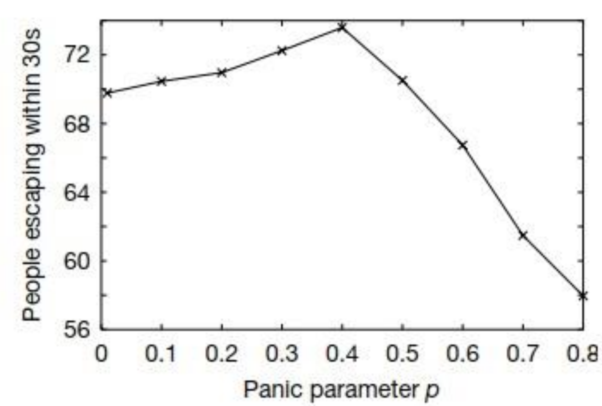
- select an individual direction $\boldsymbol{e_i}$
- follow the average direction $\langle \boldsymbol{e_j}^0(t) \rangle_i$ of his neighbors j in a certain radius R_i
- mix the two with a weight parameter p_i

$$\mathbf{e}_i^0(t) = \mathrm{Norm}[(1 - p_i)\mathbf{e}_i + p_i \langle \mathbf{e}_j^0(t) \rangle_i]$$

- if p_i is small → individualistic behavior
- if p_i is big → herding behavior
- → p_i is the "panic parameter" of individual i
- Best chances of survival: a certain mixture of individualistic and herding behavior







Faster is slower in pedestrian evacuation

Experiment (by GranularLab)

Illustrative video experimentally demonstrating the Faster is Slower effect in pedestrian evacuation through narrow doors. The charts appearing in the vertical direction are spatio-temporal diagrams constructed by taking the lines of pixels displayed by green and stacking them vertically as time evolves. For more information: http://journals.aps.org/pre/abstract/...

https://www.youtube.com/watch?v=q-k4fCiiMlk