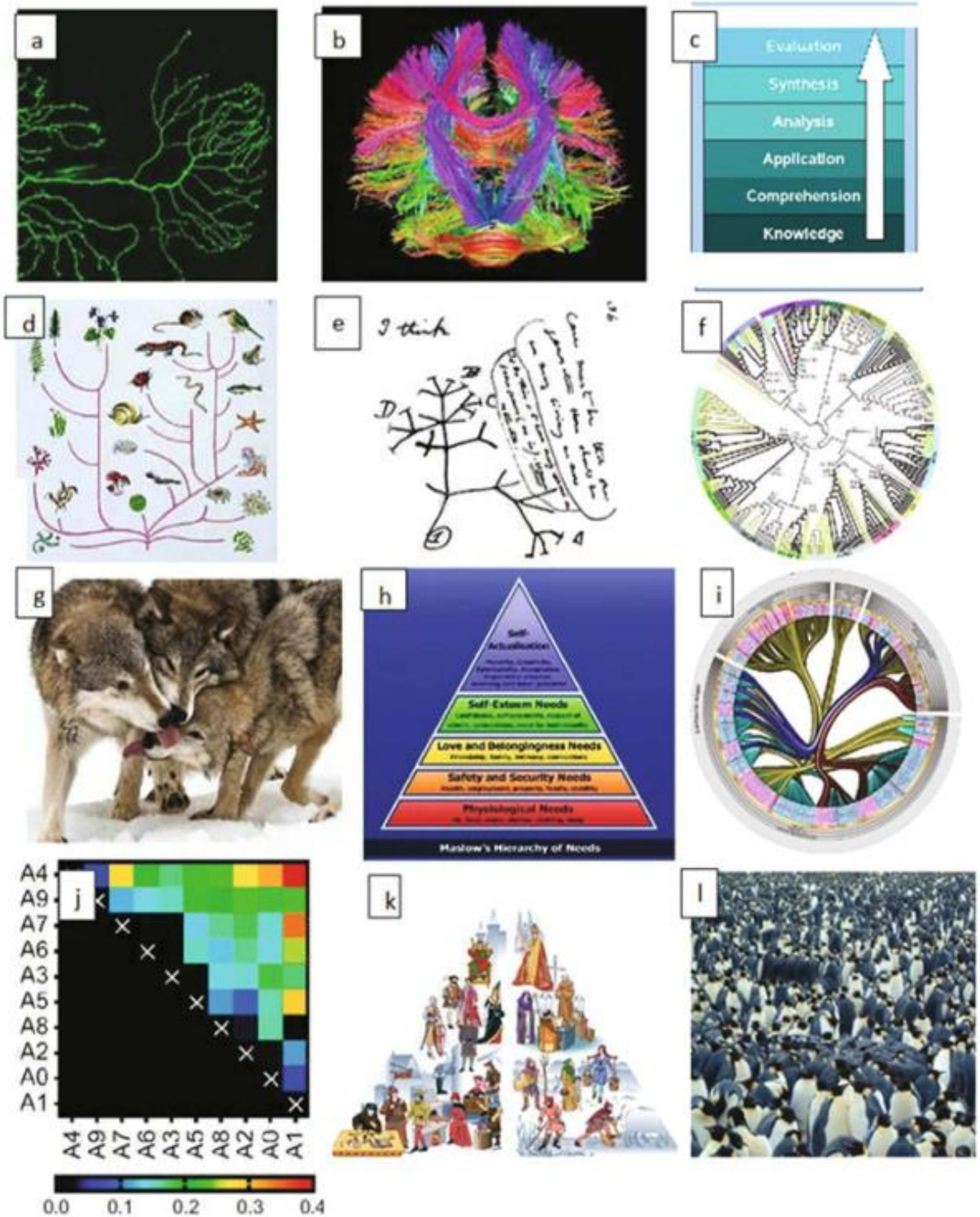
Hierarchy formation
and
collective decision making



a Axon arborisation (the end part of a major kind of neuronal cell) shows a typical hierarchical tree-like structure in space.

b The wiring of a human brain. Hierarchy is not obvious, but closer inspection and additional MRI images indicate hierarchical functional operation.

c And this is a possible interpretation of how we think (thoughts being one of the end products of a functioning brain).

d The visualization (of the now commonplace) idea of the evolutionary tree.

e The famous first drawing of the branching of the phylogenetic tree with the "I think" note by Darwin.

f This complex tree with its hundreds of branches shows the birth of new variants (associated with new plant species) of a single protein!

g The well-known hierarchy of wolves, indicated by who is licking who (subordinates do this to those above them). The same behavior can be observed between a dog and her owner.

h Perhaps the only hierarchy named after a person. This pyramid is called "Maslov's hierarchy of needs".

i Visualization of the connections (call relations) between the various parts of a C+ software system (containing many thousands of entities and relations; the more closely related parts are color-coded and bundled).

j The strength of the directional correlations between pairs of pigeons in a flock (individuals being denoted by A0,…,A9). The asymmetric structure of the dominant part of the matrix (the entire matrix minus its symmetric components) indicates strictly hierarchical leader-follower relations.

k The picturesque representation of the two pyramids of medieval relations among the member s of a society: the left side corresponding to social organization, the right side corresponding to the religious organization.

l And finally: we show a huge community of relatively simple animals. Where is the hierarchy here? Nowhere: groups of many thousands of animals (large flocks of birds, schools of fish) typically do not display the signs of hierarchy (and, indeed, are assumed not to be hierarchically organized).

Definition

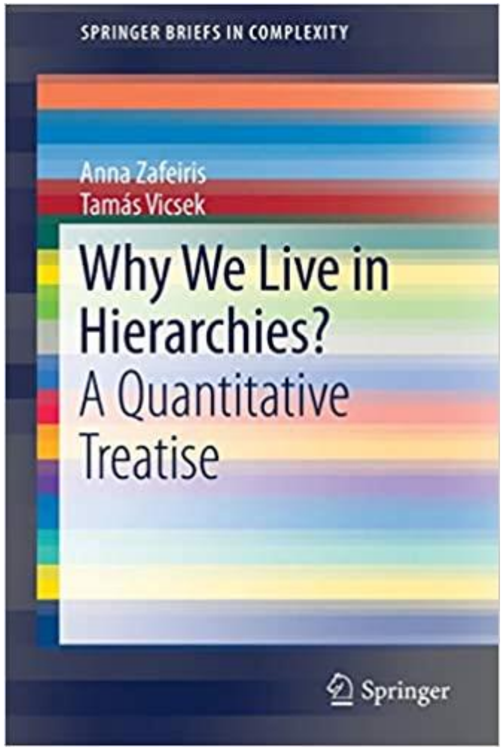
- No compact, precise, widely accepted definition
  (diverse usage)
- Available definitions usually bypass the problem of clarification by using synonymous words

- Cambridge dictionary:
  - Hierarchy is “a system in which people or things are arranged according to their importance.”
  - hierarchy corresponds to “the people in the upper levels of an organization who control it.”

- Wikipedia: “A **hierarchy** (from the Greek *hierarkhia*, "rule of a high priest", from *hierarkhes*, "president of sacred rites") is an arrangement of items (objects, names, values, categories, etc.) in which the items are represented as being "above", "below", or "at the same level as" one another.”

Definition: hierarchy

We talk about *hierarchy* in situations in which the *entities of a system can be classified into levels in a way that elements of a higher level* ***determine or constrain the behavior and/or characteristics*** *of entities in a lower level.* That is, at the heart of hierarchy, we find control of behavior.

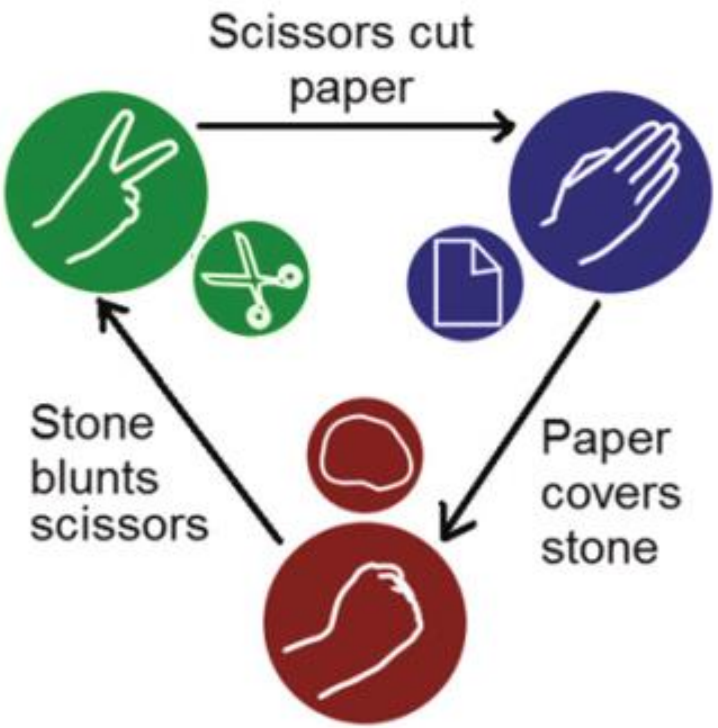


Definition: A system is *hierarchical* if it has elements (or subsystems) that are in dominant-subordinate relation to each other. A unit is *dominant* over another unit to the extent of its ability to influence the behavior of the other. In this relationship, the latter unit is called *subordinate*.

Comments on the definition of hierarchy - I

- It does not tell us how hierarchical the entire *system* is.
- It tells whether the *elements* (or subsystems) are in hierarchical relation or not? (manifesting itself in a dominant-subordinate relationship)
- It also tells the *origin* (reason) and *extent* of the dominant-subordinate relationship
- Rock–paper–scissors game:
  - The rock blunts the scissors (and hence "disarms" it, beats it)
  - The scissors cut the paper, and
  - The paper covers the rock.

- From a graph-theoretical point of view: where to put the arrows and what they mean there.
- It does not tell us how hierarchical the entire system is.
- "Measuring the level of hierarchy" in directed graphs has an entire literature



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Comments on the definition of hierarchy - II

- This definition implies that the units *behave* somehow, or have some observable characteristics. → entities without observable behavior or characteristics cannot form hierarchical structure.

- Hierarchy might vary over time.
  - As certain characteristics of the group members change (for example, the physical strength of the individuals in a pack of wolves), so do their ranks.

- During different group activities, the influence of the members might vary.

  → hierarchy is context/task-sensitive, even within the same group!
  - E.g.: pigeon flocks: Feed / collective flights.
  - even more starkly expressed in human groups

- The influence can either be
  - ***forced*** by the higher-ranked individual (e.g., when a higher-ranked animal does not let a lower-ranked one near the food source), or it can be
  - ***voluntary*** (for example, leader-follower relationships during flight).

- A higher-ranked unit, by influencing the behavior of other units more extensively, has a larger effect on the collective (emergent) group behavior as well.

Types of hierarchies

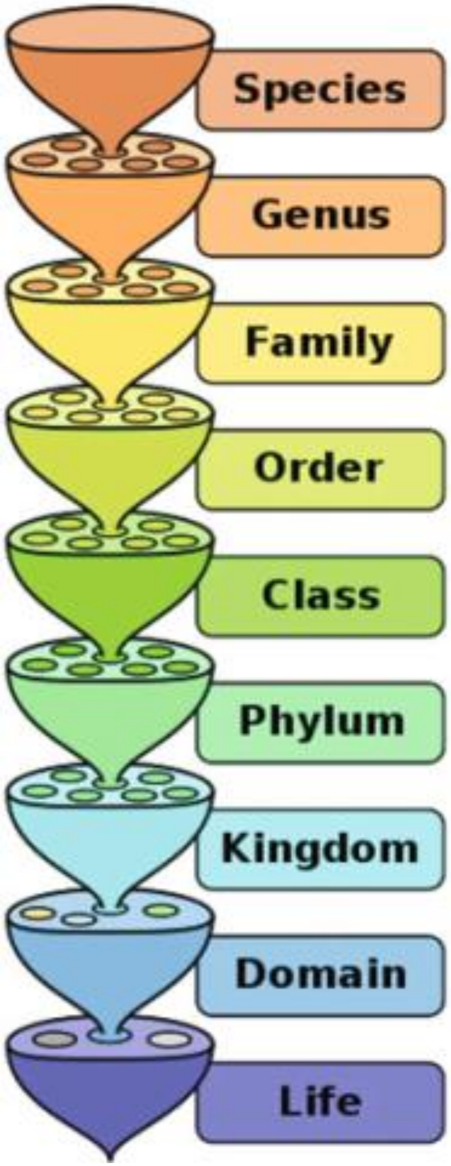
Name	Description	example

Order hierarchy	Basically an ordered set, in which a value is assigned to each element characterizing one of its arbitrarily chosen features, which defines its rank.
	
	The network behind the system is neglected or it does not exist.
		• ranking of artists, e.g., painters or sculptors, based on the average price of their artworks
		• firms ordered by their
		  • number of employees
		  • annual income, etc.




Types of hierarchies

Name	Description
Nested **Embedded** **Containment** **Inclusive** Hierarchy	A structure in which entities are embedded into each other. Higher level entities consist of and contain lower level entities. Close relation to community detection in graphs
A ***subsumptive* containment** hierarchy (a.k.a. ***taxonomic hierarchy***)	A structure in which items are classified from specific to general



C. Lupus (d dog)

Canis

Canidae

Carnivora

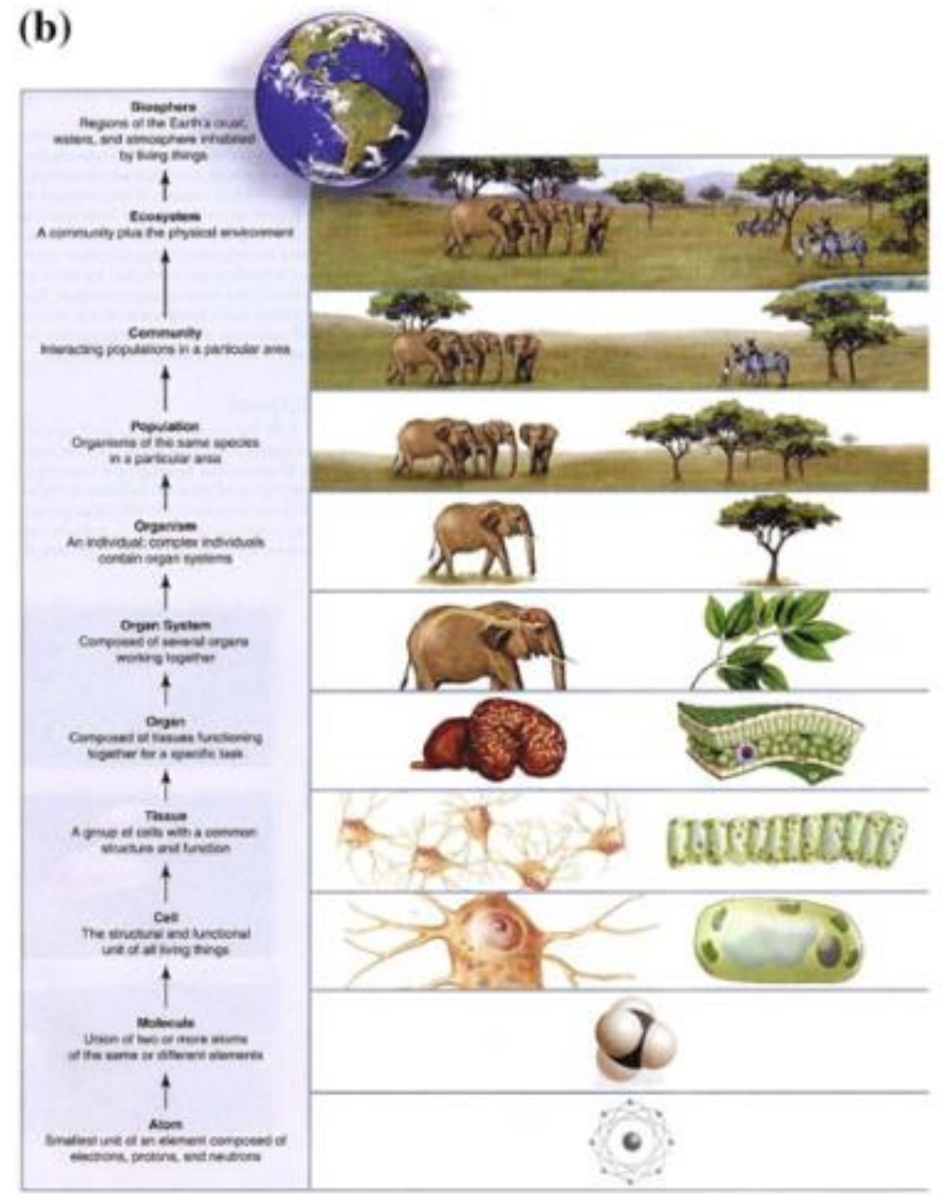
Mammalia

Chordata

Animalia

Types of hierarchies

Name	Description
Nested **Embedded** **Containment** **Inclusive** Hierarchy	A structure in which entities are embedded into each other. Higher level entities consist of and contain lower level entities. Close relation to community detection in graphs
A ***Compositional*** **containment** hierarchy ***(***a.k.a. ***level hierarchy)***	Describes how a system is composed of subsystems, which are also composed of subsystems, etc. • “Hierarchy of life”



Types of hierarchies

Name	Description	example
Flow (or **control**) hierarchy	"Intuitively," this is an acyclic, directed graph. Nodes are layered into levels: nodes on higher levels influence nodes on lower levels, and the influence is represented by edges. Layers refer to power, that is, an entity on a higher level gives orders or passes on information to entities on lower levels. ("flow of order") How certain entities control other entities.	• Armies, churches, schools, political parties, institutions, etc. • Downwards: orders flow along the edges; • Upwards: requests or information.

- These types are not independent of each other
- many systems can be described by more than one type (e.g. army: flow & compositional containment)
- Both order and nested hierarchies can be converted into a flow hierarchy.

Describing hierarchical structures

- Most commonly used mathematical tool: ***graphs***

- Primarily they are connected to systems embodying *flow hierarchy*
  - observations, experiments, computer simulations are likely to return flow hierarchy;
  - all other hierarchy types can be transformed into flow hierarchy in a rather straightforward way

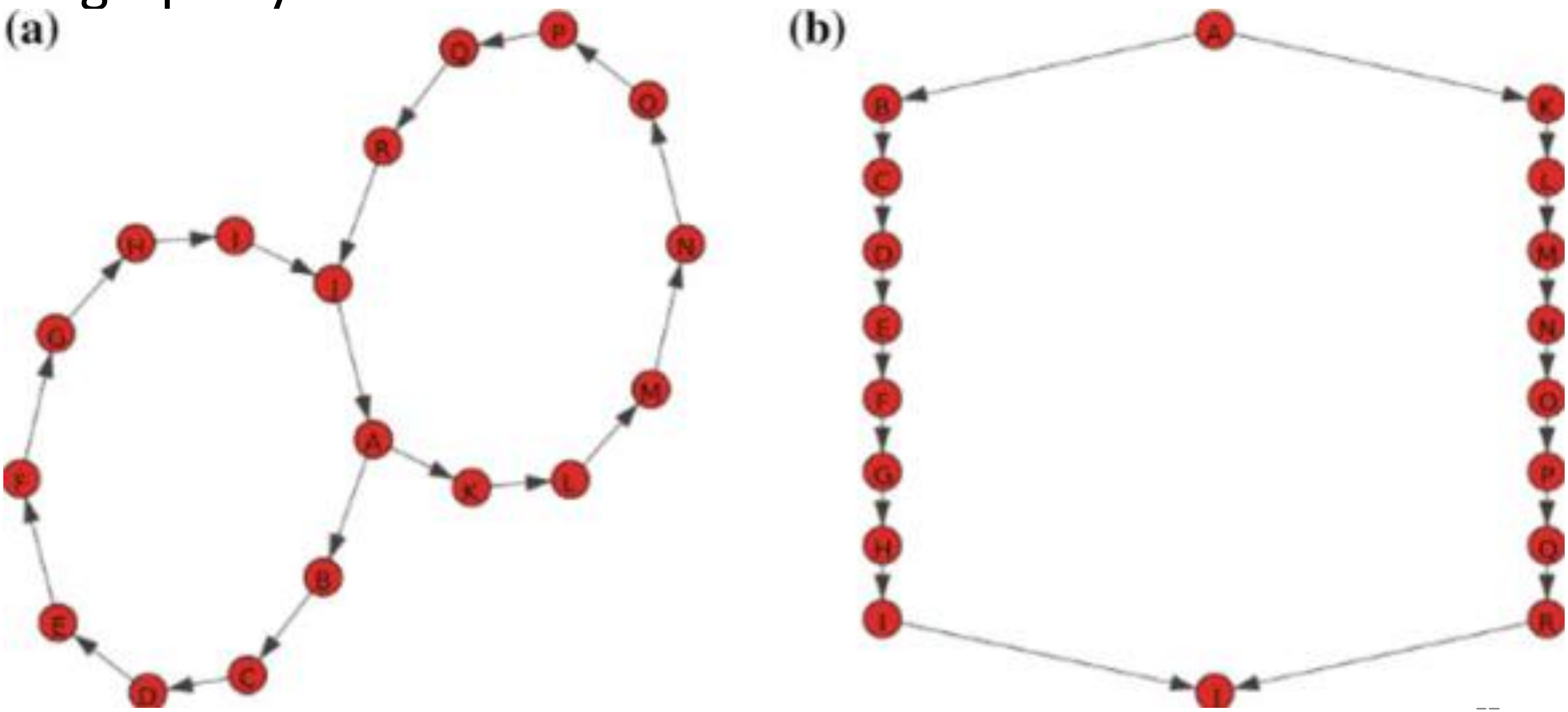
- We can measure the hierarchical level of the *graph* (not the system itself)

- No “most appropriate” measure (many structure is “matter of intuition / taste”)

- Most of the proposed measures take values on the [0, 1] interval

Some common approaches
For directed and undirected graphs

- Fraction of edges participating in cycles
- Minimum fraction of edges to be removed to make the graph cycle-free



Random Walk Measure

- Motivation:
  - it is not correct to treat all directed acyclic graphs as already being maximally hierarchical, independent of their inner structure.
  - common intuition: a hierarchical structure often corresponds to a multi-level pyramid in which the levels become more and more wide as one descends from the higher levels towards the lower ones
- Assumption: there is information/instruction flow from the high-ranking nodes towards the bottom ones
- Method:
  - find the sources by dropping down random walkers onto the nodes who then move *backwards* along the links
  - Once a steady state is reached, the *density* of such random walkers is interpreted as being proportional to the *rank* of the node:
    - high random walker density: the vertex is a *source* of information (high rank)
    - low density: the vertex is just a “receiver” of orders (low rank)
  - The hierarchical nature of the network: estimated based on the *distribution* of random walker densities
    - Homogeneous: the source of information/order cannot be pinpointed: not hierarchical
    - Inhomogeneous: clear information sources: the network is hierarchical.

Czégel D, Palla G (2015) *Random walk hierarchy measure: what is more hierarchical, a chain, a tree or a star?* Sci Rep 5:17994

Global Reaching Centrality ("GRC")

- Central idea: to give a rank to each node by measuring its "impact" on other nodes
  - "Impact": the ratio of vertices that can be reached from the focal node i – this is the *"local reaching centrality"*
  - In a directed, un-weighted graph $C_R(i)$ is the number of vertices that can be reached from node i, divided by $N-1$
  - The level of hierarchy is inferred from the *distribution* of the $C_R(i)$ values
    - Heterogeneous distribution: hierarchical network
    - Homogeneous distribution: non-hierarchical graph
- From distribution to number:
  - Let C_R^{max} denote the highest $C_R(i)$ value in a graph $G=(V,E)$
  - Then *GRC*, the Global Reaching Centrality is:

$$\mathrm{GRC} = \frac{\sum_{i \in V} \left[C_R^{\max} - C_R(i) \right]}{N-1}$$

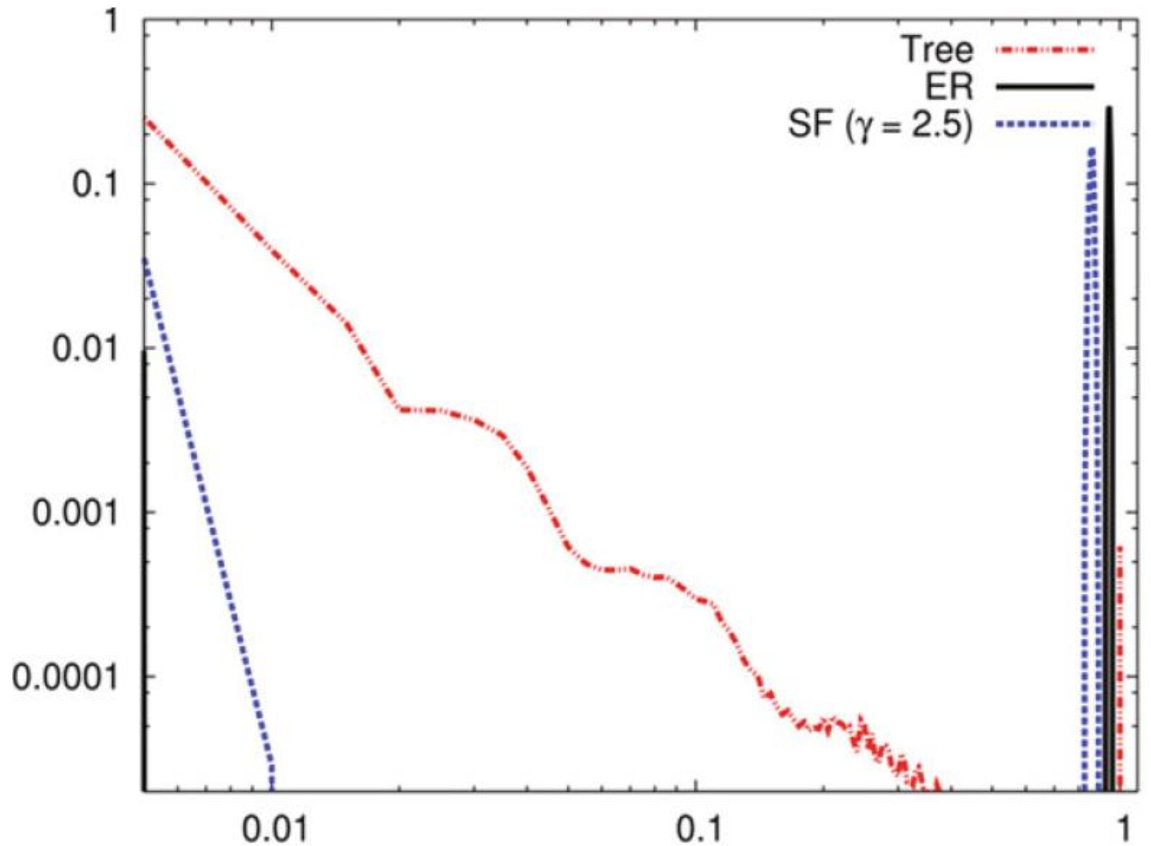
Global Reaching centrality ("GRC")

Example: GRC distribution for three different network types:

- Erdős-Rényi (random) (not hier)
- Scale-free (moderately hier)
- Tree (highly hier)

$$\mathrm{GRC} = \frac{\sum_{i \in V} \left[C_R^{\max} - C_R(i) \right]}{N-1}$$

Network type	GRC
Erdős-Rényi	0.058 ± 0.005
Scale-free	0.127 ± 0.008
Tree	0.997 ± 0.001

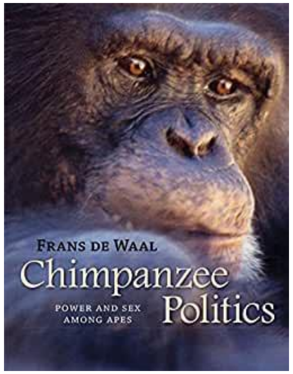


Distributions of the local reaching centralities for three kinds of directed network: Tree, Erdős-Rényi (ER) and scale-free (SF). All the curves are averages of *1000* graphs with *N = 2000*, of the appropriate graph type.

Observations and measurements

Dominance hierarchy

- Solitary vs. social lifestyles
- If the ratio of advantages/disadvantages is higher, then the given animals will knit into groups
- A mechanism is needed to reduce the level of aggression triggered by the competition
- Regulate access to resources.
- The mechanism is simple: higher ranked individuals have primacy compared to their lower level mates.
- As one advances in the evolutionary tree, the structure of the dominance hierarchy gets more and more pronounced and complex, accompanied by more and more sophisticated strategies by which individuals try to get higher and higher ranks.
- Chimpanzees (few decades ago believed to be solely human):
  - coalition formation
  - manipulation
  - exchange of social favors
  - adaptation of rational strategies
- Obvious advantage: less fight



Leadership in motion

The relation of collective motion to collective decision making

- If the group is to stay together, individuals constantly have to make decisions regarding
  - When and where to forage, to rest
  - How to defend themselves from predators
  - How to navigate towards a distant targets
  - Etc.
- Cost/benefit ratio (from the viewpoint of the members)
  - Preferred outcome usually differs (information, experience, inner state, etc.)
  - "consensus cost": cost paid by the animal who foregoes its preferred behavior in order to defer to the common decision

First studies – two basic types

Despotic system

- One or a few individual decides
- This can increase the efficiency

Egalitarian / democratic

- Members contribute to the outcome about the same degree
- Smaller average consensus cost

- In nature, both types have been observed
- Sometimes mixed (alternating according to the circumstances)
  - Pairs of pigeons, GPS (2006)
    - Small conflict over the preferred direction: consensus (average)
    - Above a certain threshold: one of them becomes the leader or they split up
  - Similar observations: Wild baboons, GPS (2015)
    - They follow the majority of the “initiators” (those starting off in a certain direction). (And not the dominant individuals)
    - If two groups of initiators (with similar size) heading in different directions:
      - If the angle is less than ~90° → the animals compromise
      - Big angle: they choose one direction over the other (randomly)

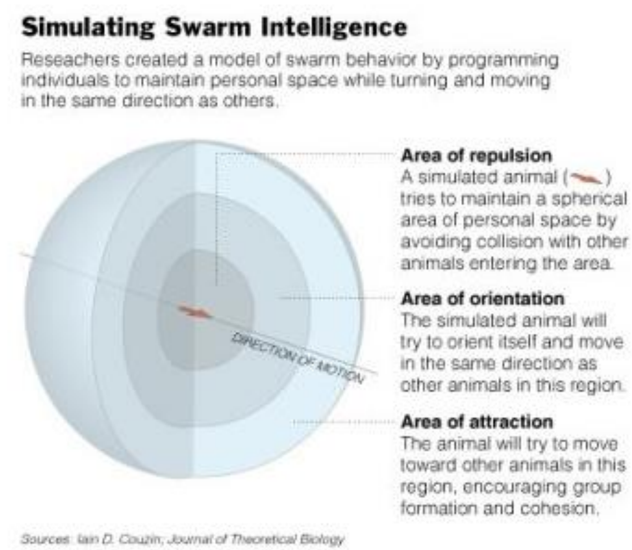
Models for leadership

- Extension of the "Couzin model"
- No individual recognition, no signaling mechanism
- Non-informed individuals: are not required to know how many and which individuals has information
- Vice versa: Informed individuals are not required to know anything about the information-level of their mates and that how the quality of their information was compared to that of others.

The model:

- Rule 1: highest priority
  - Individuals attempt to maintain a certain distance among themselves by turning away from those neighbors j which are within a certain distance towards the opposite direction:

$$\vec{d}_i(t+\Delta t) = -\sum_{j\neq i} \frac{\vec{r}_j(t)-\vec{r}_i(t)}{\left|\vec{r}_j(t)-\vec{r}_i(t)\right|}$$



$\vec{d}_i$: desired direction of individual i
$\vec{r}_i$: position of particle i
$\vec{v}_i$: direction of unit i

[Couzin, I.D., Krause, J., Franks, N.R., Levin, S.A., 2005. Effective leadership and decision-making in animal groups on the move. Nature 433, 513–516.]

Models for leadership

The model (cont):

- Rule 2

If there are no mates within the range of repulsion, than the individual will attempt to align with those neighbors j, which are within the range of alignment:

→ The desired direction:

$$\vec{d}_i(t+\Delta t) = -\sum_{j\neq i}\frac{\vec{r}_j(t)-\vec{r}_i(t)}{\left|\vec{r}_j(t)-\vec{r}_i(t)\right|} + \sum_{j\neq i}\frac{\vec{v}_j(t)}{\left|\vec{v}_j(t)\right|}$$

$\vec{d}_i$: desired direction of individual i
$\vec{r}_i$: position of particle i
$\vec{v}_i$: direction of unit i

- Corresponding unit vector: $\hat{d}_i(t) = \vec{d}_i(t)/\left|\vec{d}_i(t)\right|$
- Introducing "influence": a portion of the group (p) is given information/motivation about a preferred direction, described by the (unit) vector $\vec{g}$.
- The rest of the group does not have directional preference.

Informed individuals balance their

- social alignment $\hat{d}_i(t)$ (the unit vector of $\vec{d}_i(t+\Delta t) = -\sum_{j\neq i}\frac{\vec{r}_j(t)-\vec{r}_i(t)}{|\vec{r}_j(t)-\vec{r}_i(t)|} + \sum_{j\neq i}\frac{\vec{v}_j(t)}{|\vec{v}_j(t)|}$) and
- preferred direction $\vec{g}_i$

with the weighting factor ω:

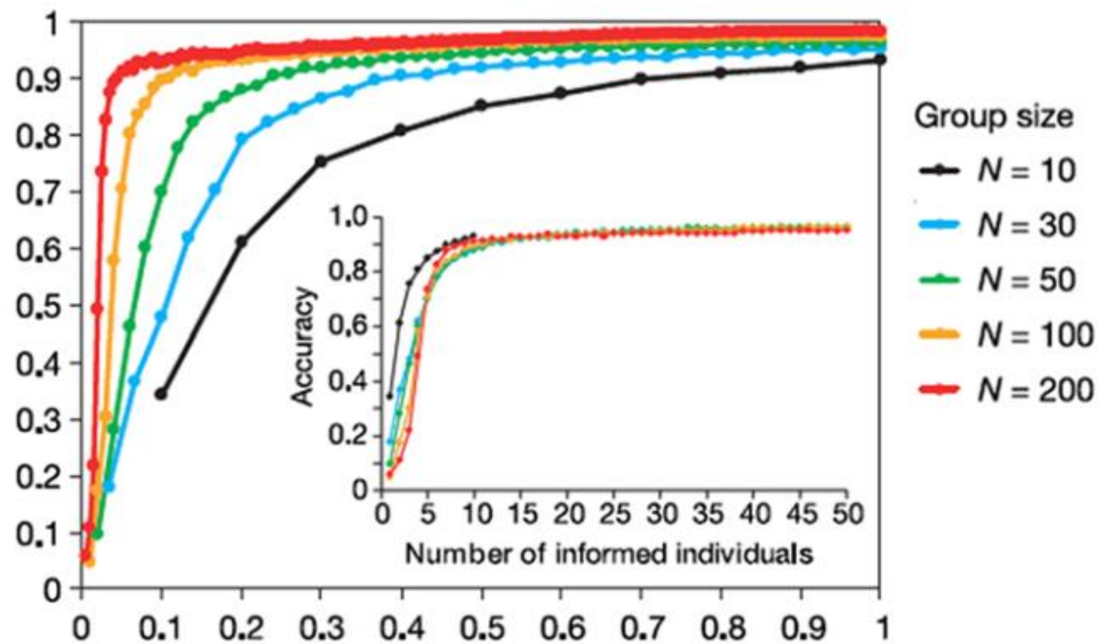
$$\vec{d}_i(t+\Delta t) = \frac{\hat{d}_i(t+\Delta t) + \omega\vec{g}_i}{|\hat{d}_i(t+\Delta t) + \omega\vec{g}_i|}$$

- ω can exceed 1: the individual is influenced more by its own preferences than by its mates
- "Accuracy" of the group: normalized angular deviation of the group direction around the preferred direction $\vec{g}_i$

Results:

- for fixed group size, the accuracy increases asymptotically as the portion p of the informed members increases

  (…that is…)
- the larger the group, the smaller the portion of informed members is needed, in order to guide the group towards a preferred direction



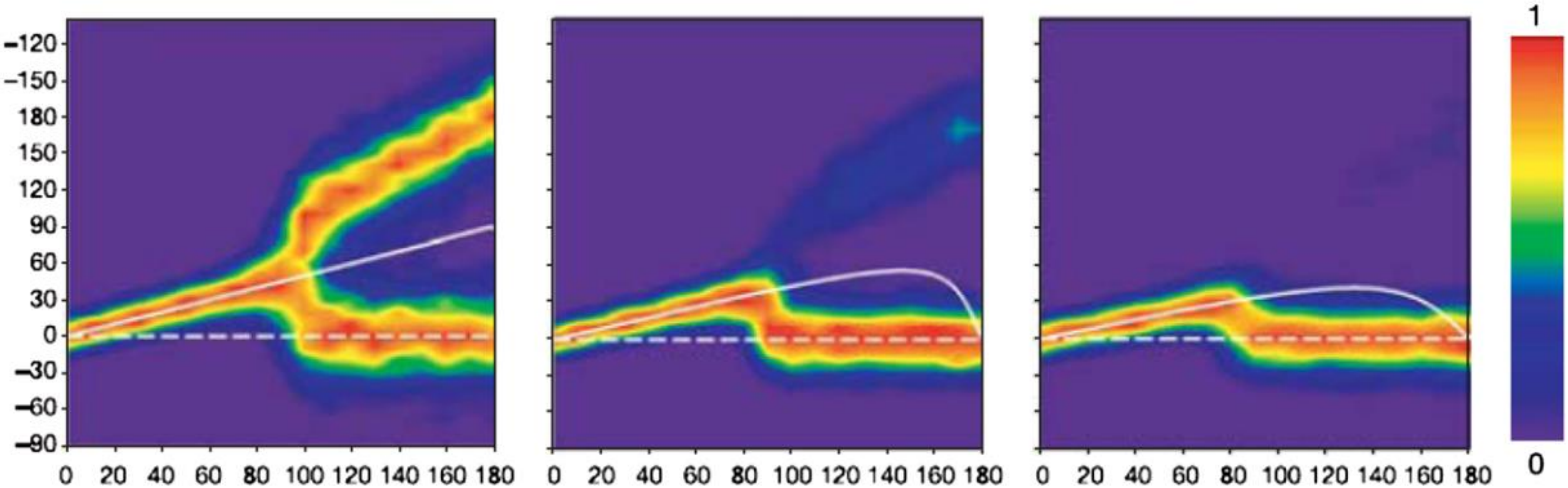
Conflicting preferences

Informed individuals might differ in their preferred direction

1. If the number of individuals preferring one or another direction is equal: the group direction depends on the degree to which the preferred directions differ
   - If it is small: the group will go in the average preferred direction of all informed individuals
   - If it is big: individuals select randomly one or another preferred direction

2. If the number of informed individuals preferring a given direction increases
   - the entire group will go into the direction preferred by the majority (even if that majority is small)

Collective group direction when two groups of informed individuals differ in their preferences - model results



$n_1 = n_2 = 5$

$n_1 = 6$
$n_2 = 5$

$n_1 = 6$
$n_2 = 4$

- Vertical axis: the degree of the most probable group motion.
- The first group (consisting of n_1 informed individuals) prefers the direction characterized by 0 degrees (dashed line),
- The second group (consisting of n_2 informed individuals) prefers a direction between 0 and 180 degrees (horizontal axis)

- Solid white lines are for reference only, representing the direction of the average vector of all informed individuals
- The group consists of 100 individuals altogether

Source: Couzin, I.D., Krause, J., Franks, N.R., Levin, S.A., 2005. Effective leadership and decision-making in animal groups on the move. Nature 433, 513–516.

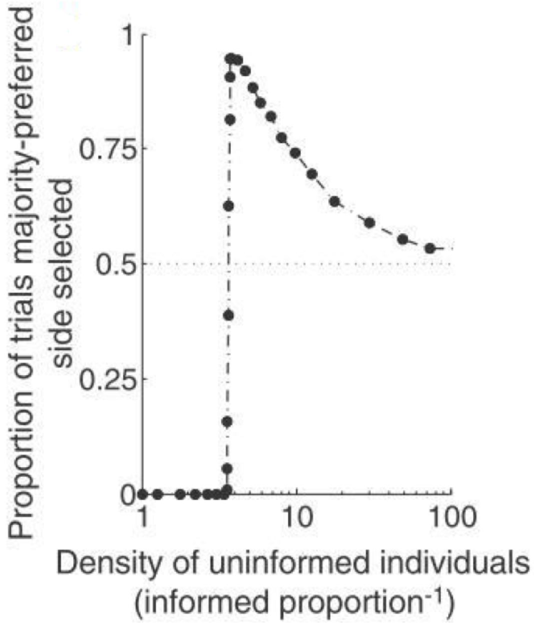
The role of uninformed individuals – simulations vs. experiments

- **Question**: under what conditions can a self-interested and strongly opinionated minority exert its influence on group movement decisions?

- Simulations:
  - Based on the "Couzin model"

$$\vec{d}_i(t+\Delta t) = \frac{\hat{d}_i(t+\Delta t) + \omega \vec{g}_i}{\left|\hat{d}_i(t+\Delta t) + \omega \vec{g}_i\right|}$$

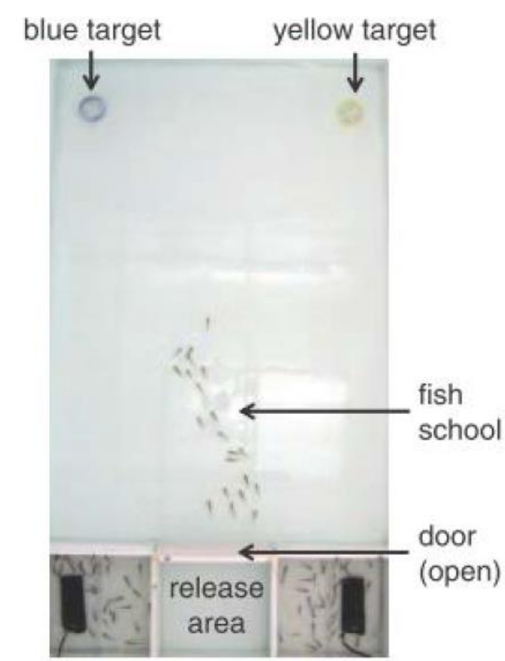
  - If all individuals are biased:
    - If the strength of the majority preference (ω_1) is equal to or stronger than the minority preference (ω_2), the group has a high probability of reaching the majority-preferred target.
    - Increasing ω_2 (beyond ω_1) can result in the minority gaining control
  - If there are uninformed individuals ($\omega_3 \approx 0$):
    - (most animal groups are like this)
    - Adding uninformed individuals tends to return control spontaneously to the numerical majority
    - this effect reaches a maximum and then begins to slowly diminish, and eventually, noise will dominate



A sharp transition from a minority- to majority-controlled outcome in the model as the density of uninformed individuals is increased. ($\omega_{minority} > \omega_{majority}$)

Experiment

- golden shiners
- two groups of initiators (with sizes N_1 and N_2) with different preferred directions (blue and yellow target)
- some did not have direction preference
- $N_1 > N_2$ ($N_1 = 6$ and $N_2 = 5$)
- Among the trained fish, ω_{yellow} is "by nature" $> \omega_{blue}$
- Simulations predict a large effect for a relatively small number of naïve individuals; $N_3 = 0, 5, 10$.
- When all individuals exhibit a preference ($N_3 = 0$) then the minority N_2 dictates the consensus (even though the fish trained to the blue target are more numerous).
- When untrained individuals are present, they increasingly return control to the numerical majority N_1.
- If individuals with the stronger preference were also in the numerical majority: the majority was more likely to win (72% of trials overall), and the presence of uninformed individuals had no effect



Experimental set-up

Couzin et al, 2011, Uninformed individuals promote democratic consensus in animal groups. Science, 334(6062):1578-80

Lessons

- Leadership might emerge from the differences of the level of information possessed by the group members
- information can be pertinent → leadership can be transient and transferable too