Opinion dynamics

Opinion dynamics

- The scientific field aiming to understand the way „opinions” spread in human communities.
  - The community is usually described by means of networks
    - Nodes are individuals
    - Links are the ties (connections)
      - Direction
      - Strength
  - „opinion” (or „state” of the node) is usually described by a scalar (binary or continuous)

- Close relation to fields studying other spreading phenomena
  - Infection spreading

- Relevant general questions include:
  - What are the fundamental interaction mechanisms („local rules”) that allow for the emergence of
    - Consensus / polarization / fragmentation
    - a shared culture
    - a common language, etc. …
  - What favors the homogenization process? What hinders it?

- “Unfortunately”: Opinion formation is a complex process affected by the interplay of different elements, including the
  - Individual predisposition / family background
  - Background knowledge
  - External information (e.g. public media)
  - Etc.

Typical models

- Consider a finite number of connected agents
- each possessing opinions, described by variables,
- Assume certain *local rules* by which opinions change
  - Change of opinions result from interactions, either with peers or other sources.

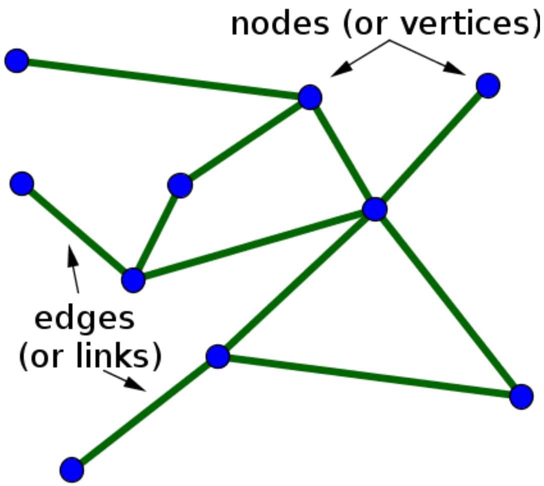
- Opinions:
  - Variables:
    - one dimensional/multidimensional vector
    - discrete (the components can assume a finite number of states)
    - or continuous (values in the domain of real numbers)

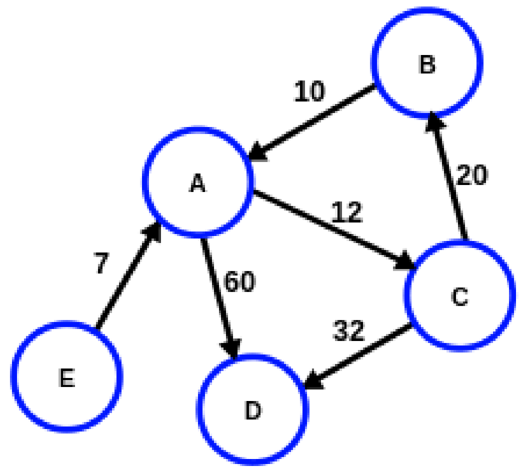
- Connections:
  - Topology of the interaction NW (what is realistic?)
  - "Heritage" from physics: lattices or all-to-all (MF);
    (hardly realistic in social context)

- Drawbacks of the models:
  - many simplifications;
  - many of the omitted parameters (most probably) have a fundamental effect in the final dynamics
  - Hard to say when are the results „good” (polarization)

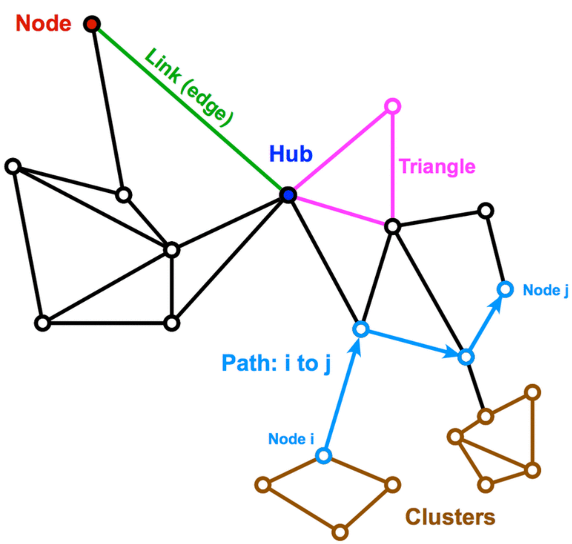
- (It is said to have) Success in:
  - Agreement
  - Cluster formation
  - Transition between order (consensus) and disorder (fragmentation)

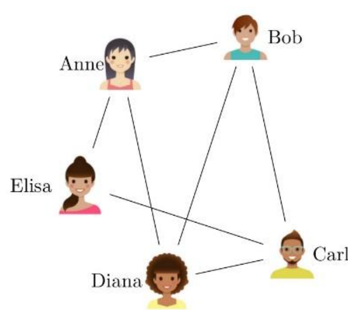
Basic concepts of networks

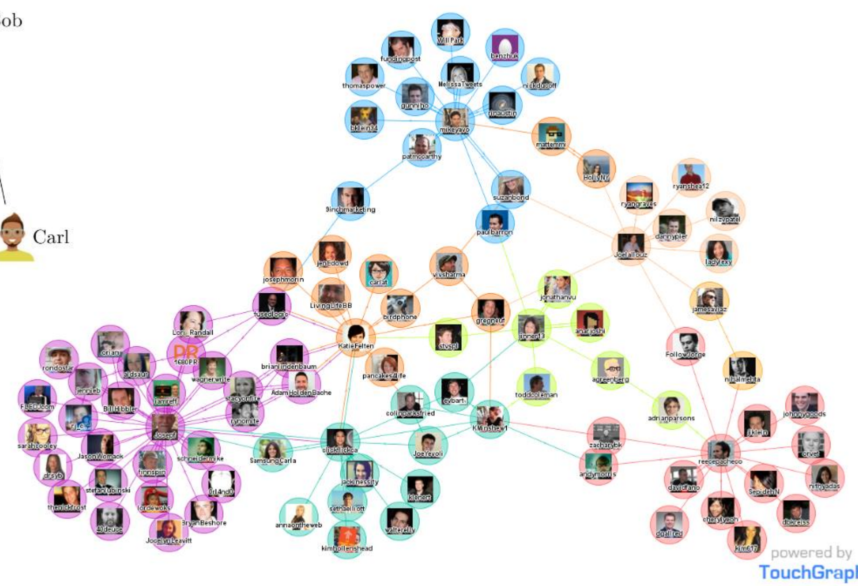




With some network analysis







Binary opinions

- Discrete, one dimensional
- 0/1; yes/no; etc
- Interpretation in op. dyn: political questions

  infection models: infected / not

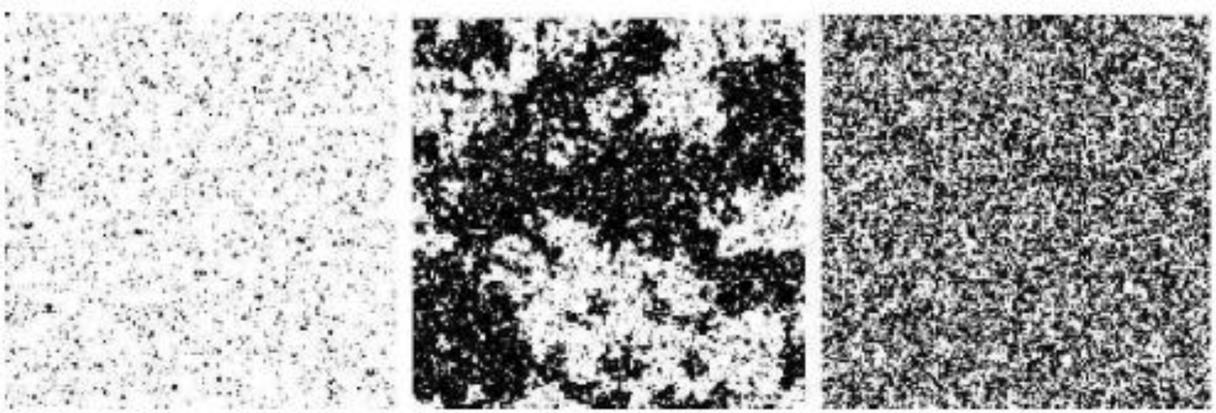
  market behavior: selling/buying

Very first opinion dynamic model by physicist:
1971, Weidlich

Ising model metaphor

- Consider a collection of N spins (agents): s_i
- They can assume two values: +/- 1
- Each spin is energetically pushed to be aligned with its nearest neighbors.
- The total energy is:
  (the sum runs on the pairs of nearest-neighbors) $H = -\frac{1}{2} \sum_{<i,j>} s_i s_j$

- Elementary move:
- a single spin flip is accepted with probability $\exp(-\Delta E / k_B T)$
  - ΔE: change in the energy
  - T: temperature (In ferromagnetic systems thermal noise injects fluctuations – tends to destroy order)
  - Critical temperature T_c : above: the system is macroscopically disordered
    under: long-range order is established



Snapshots of equilibrium configurations of the Ising model (from left to right) below, at and above T_c.

Relation to opinion dynamics models

- Each agent has one opinion represented as a spin:

  a choice between two options
- Spin couplings: peer interactions (social conformity)
- Magnetic field: external information / propaganda
- Simple, but attractive model

Potts model (1951)

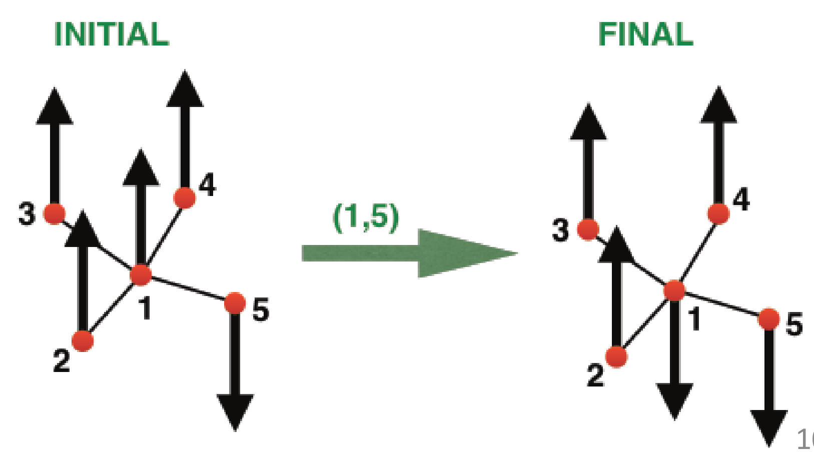
- a generalization of the Ising model
- Each spin can assume one out of q values
- equal nearest neighbor values are energetically favored.
- The Ising model corresponds to the special case q=2

Voter model

- Originally introduced to analyze competition among species, early 1970s
- Rather crude description of any real process
- Popular: it is one of the very few non-equilibrium stochastic processes that can be solved exactly in any dimension
- its name stems from its application to electoral competitions
- The model:
  - each agent in a population of N holds one of two discrete opinions: $s = +/- 1$
  - agents are connected by an underlying graph (topology)
  - At each time step:

a random agent i is selected (1) along with one of its neighbors j (5) and the agent takes the opinion of the neighbor: $s_i = s_j$

(alignment *not* to the majority, but to a random neighbor)

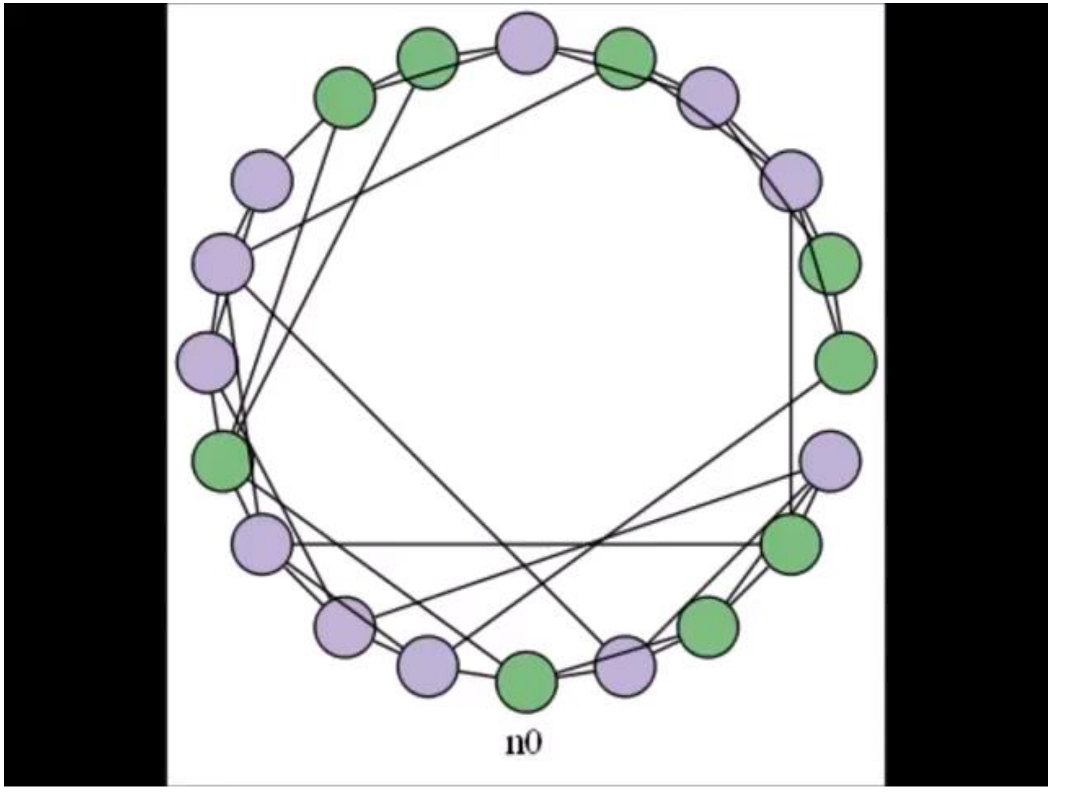


Behavior of the Voter model

- Has been extensively studied
- If people are modeled as vertices in a *d*-dimensional hyper-cubic lattice.
  - For finite system: for any dimension *d* of the lattice, the voter dynamics always leads to one of the two possible consensus states: each agent with the same opinion s = 1 or s = −1.
  - The probability of reaching one or the other state depends on the initial state of the population.
  - Time needed for reaching the consensus state:
    - d = 1: $T_N \sim N^2$
    - d = 2: $T_N \sim N \ln N$
    - d > 2: $T_N \sim N$
  - For infinite systems: consensus is reached only if d ≤ 2

Extensions of the voter model

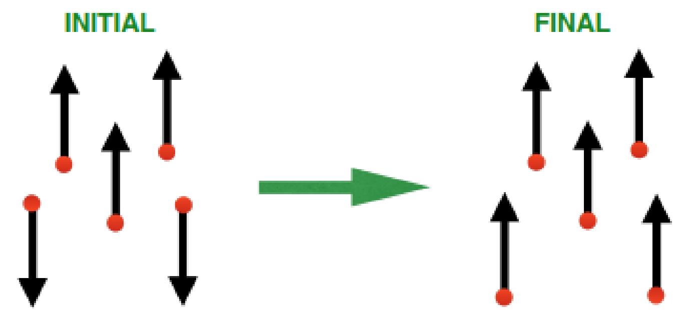
- Introduction of "zealots": individuals who do not change their opinion
- Constrained voter model:
  - agents can be leftist, rightist, centralist;
  - Extremists do not talk to each other (discrete analogue of the bounded confidence model)
- Communication is based on various NW



Voter model on a small world network
https://www.youtube.com/watch?v=VmhSTdrsimk

Majority rule model

- Motivation: describing public debates
- (Galam, 2002)
- Definition:
  - Population of N agents
  - A fraction p_+ of agents has opinion +1
  - $p_- = 1- p_+$ has opinion -1
  - Everybody can communicate with everybody else
  - At each interaction:
    - A group of r agents are selected at random (“discussion group”)
    - Consequence of this interaction: each agents take the majority opinion inside the group
  - r is taken from a given distribution at each step
    - If r is odd: there is always a clear majority
    - If r is even: in case of tie: a bias is introduced in favor of one of the options (Inspired by the principle of “Social inertia” holding that people are reluctant to accept a reform if there is no clear majority in its favor)



Basic features of the MR model

- Original definition:
  - There is a *threshold fraction* p_c such that if $p^+_0 > p_c$, then all agents will have opinion +1 in the long run
  - Time needed for the consensus: $T_N \sim \log N$
    - If the group sizes r are odd: $p_c(r) = 1/2$ (due to the symmetry)
    - If they can be even too: $p_c < 1/2$, that is, the favored opinion will eventually win, even if it was originally in minority

- For fixed odd r, group size & mean field approach: analytically solvable for both finite N and for $N \rightarrow \infty$

- Many variants and modifications

Social impact theory

- Bibb Latané (psychologist), 1981:
- social impact: any influence on individual feelings, thoughts or behavior that is created from the real, implied or imagined presence or actions of others.

  („Collective” behavior)
- The impact of a social group on a subject depends on:
  - The number of individuals within the group
  - Their convicting power
  - Their distance from the subject (in an abstract space of personal relationships)
- Originally a cellular automata was introduced by Latané (1981) and later refined by Nowak et al (1990).

Social impact theory – the model

- A population of N individuals
- Each individual i is characterized by
  - an opinion $\sigma_i=\pm 1$
  - Persuasiveness p_i : the capability to convince someone **to change** opinion (a real value)
  - Supportiveness: s_i : the capability to convince someone **to keep** its opinion (a real value)
    (these are assumed to be random)
- The distance between agents i and j d_{ij},
- α>2 parameter defining the how fast the impact decreases with the distance

$$I_i = \underbrace{\left[\sum_{j=1}^{N} \frac{p_j}{d_{ij}^{\alpha}}(1-\sigma_i\sigma_j)\right]}_{\text{Persuasive impact (to change)}} - \underbrace{\left[\sum_{j=1}^{N} \frac{s_j}{d_{ij}^{\alpha}}(1+\sigma_i\sigma_j)\right]}_{\text{supportive impact (to keep opinion)}}$$

Opinion dynamics: $$\sigma_i(t+1) = -sgn[\sigma_i(t)I_i(t)+h_i]$$

h_i : personal preference, originating from other sources (e.g. mass media)

a spin flips if the pressure in favor of the opinion change overcomes the pressure to keep the current opinion (I_i > 0 for vanishing h_i)

General behavior of the social impact model

- In the absence of individual fields (personal preferences):
  - the dynamics leads to the dominance of one opinion over the other, but not to complete consensus.
- In the presence of individual fields:
  - these minority domains become metastable: they remain stationary for a very long time, then they suddenly shrink to smaller clusters, which again persist for a very long time, before shrinking again, and so on (“staircase dynamics”).

- Many modification / extensions:
  - Learning
  - Presence of a strong leader
  - Etc.

Schweitzer and Holyst included: (2000)

- Memory: reflecting past experience
- A finite velocity for the exchange of information between agents
- A physical space, where agents move.

Continuous opinions

- In many cases more realistic
- Requires different framework
  - Concepts like “majority” or “opinion equality” don’t work
  - Has a different ‘history’
- First studies (end of 1970’s and 80’s):
  - Aimed to study the conditions under which a panel of experts would reach a common decision (“consensus”)
  - By applied mathematicians
- Typically:
  - **Initial state**: population of *N* agents with randomly assigned opinions, represented by real values within some interval.

    discrete op. dyn. ↔ all agents start with different opinions
  - **Possible scenarios**: more complex
    - Opinion clusters emerging in the final stationary state:
      - one cluster: consensus,
      - two clusters: polarization
      - more clusters: fragmentation

Bounded confidence (BC) models

- **In principle:** each agent can interact with every other
- **In practice:** (often) there is a real discussion only if the opinions are sufficiently close:

  bounded confidence
- **In the literature:** introducing a real number ε: "*uncertainty*" or "*tolerance*", such that:
- An agent with opinion *x*, only interacts with those whose opinion lies in the interval]x−ε, x+ε[
- („Homophily")

Deffuant model

- population of N agents
- nodes of a graph: agents may discuss with each other if they are connected.
- Initially: each agent *i* is given an opinion x_i randomly chosen from the interval [0, 1].
- Dynamics:
  - random binary encounters, i.e., at each time step, a randomly selected agent discusses with one of its neighbors, also chosen at random.
  - Let *i* and *j* be the pair of interacting agents at time *t*, with opinions $x_i(t)$ and $x_j(t)$
    - if the difference of the opinions $x_i(t)$ and $x_j(t)$ exceeds the threshold ε, nothing happens
    - If $|x_i(t) - x_j(t)| < \varepsilon$, then
    - μ: convergence param. (μ in [0, 1/2])

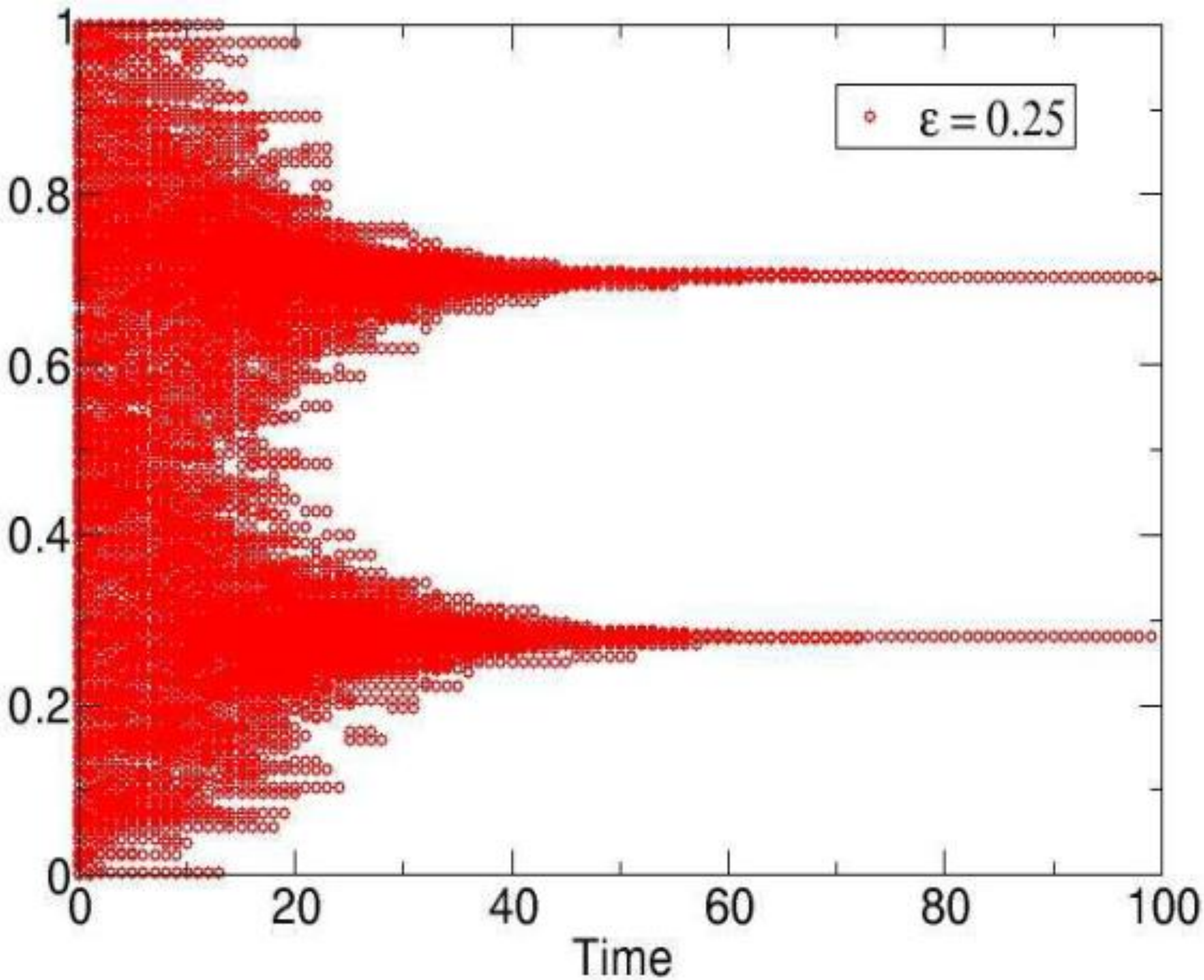
$$x_i(t+1) = x_i(t) + \mu[x_j(t) - x_i(t)]$$

$$x_j(t+1) = x_j(t) + \mu[x_i(t) - x_j(t)]$$

Behavior of the Deffuant model

- For any value of ε and μ, the average opinion of the agents' pair is the same before and after the interaction → the global average opinion (1/2) of the population is invariant
- Patches appear with increasing density of agents
- Once each cluster is sufficiently far from the others (the difference of opinions in distinct clusters exceeds the threshold):
  - only agents *inside* the same cluster interact
  - the dynamics leads to the convergence of the opinions of all agents in the cluster
- In general:
  - the *number and size of the clusters* depend on the threshold ε
    (if ε is small, more clusters emerge)
  - the parameter μ affects the convergence time
  - (when μ is small, the final cluster configuration also depends on μ)

Behavior of the Deffuant model



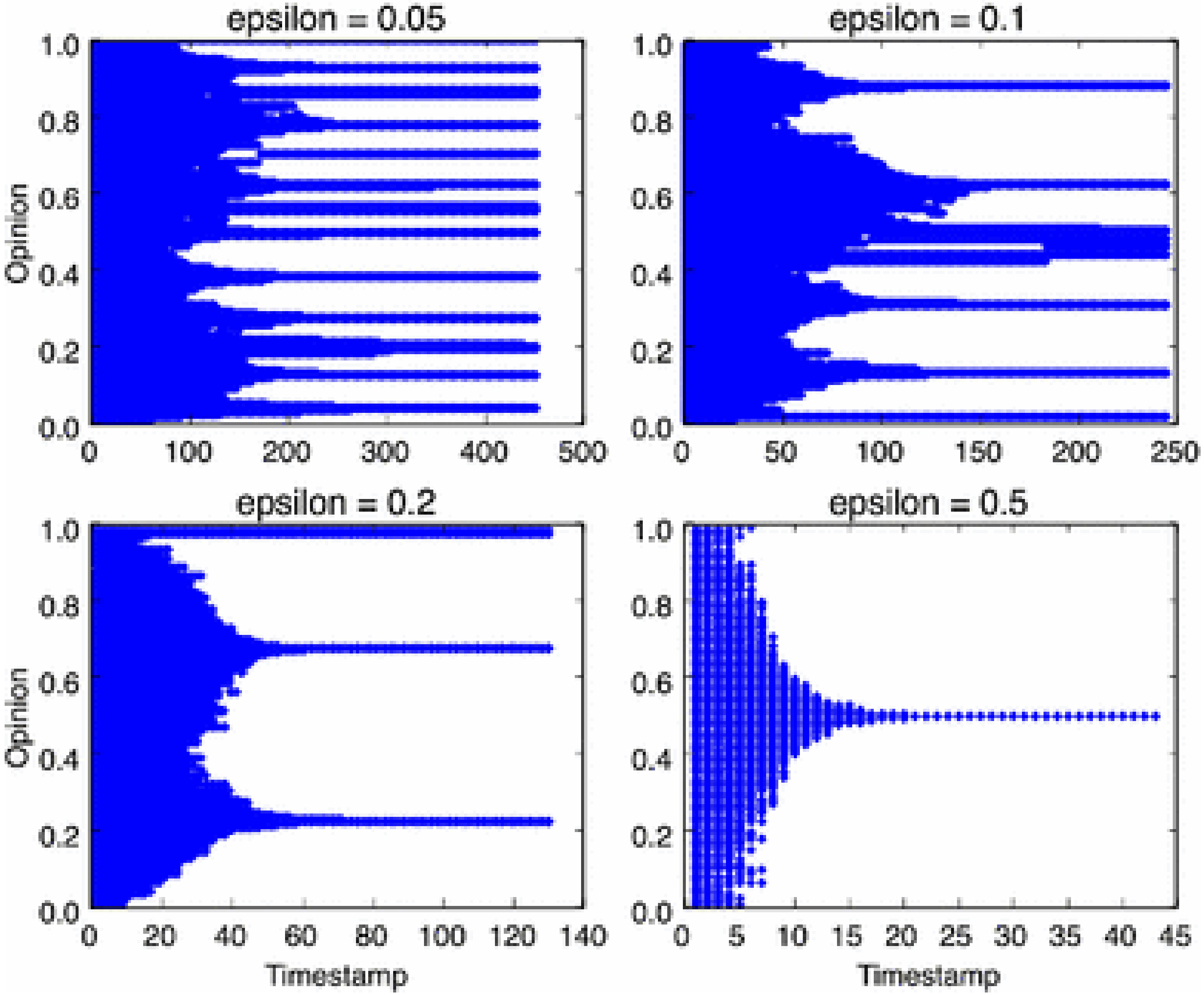
Opinion profile of a population of N=500 agents during its time evolution,

ε = 0.25.

The population is fully mixed, i.e., everyone may interact with everybody else.

The dynamics leads to a polarization of the population in two factions.

Behavior of the Deffuant model



Hegselmann-Krause (HK) model

- Hegselmann and Krause, 2002

- Similarities with the Deffuant model:
  - Opinions take real values in an interval, say [0, 1]
  - An agent *i* (with opinion x_i), interacts with neighboring agents whose opinions lie in the range $[x_i - \varepsilon, x_i + \varepsilon]$
- Difference: update rule

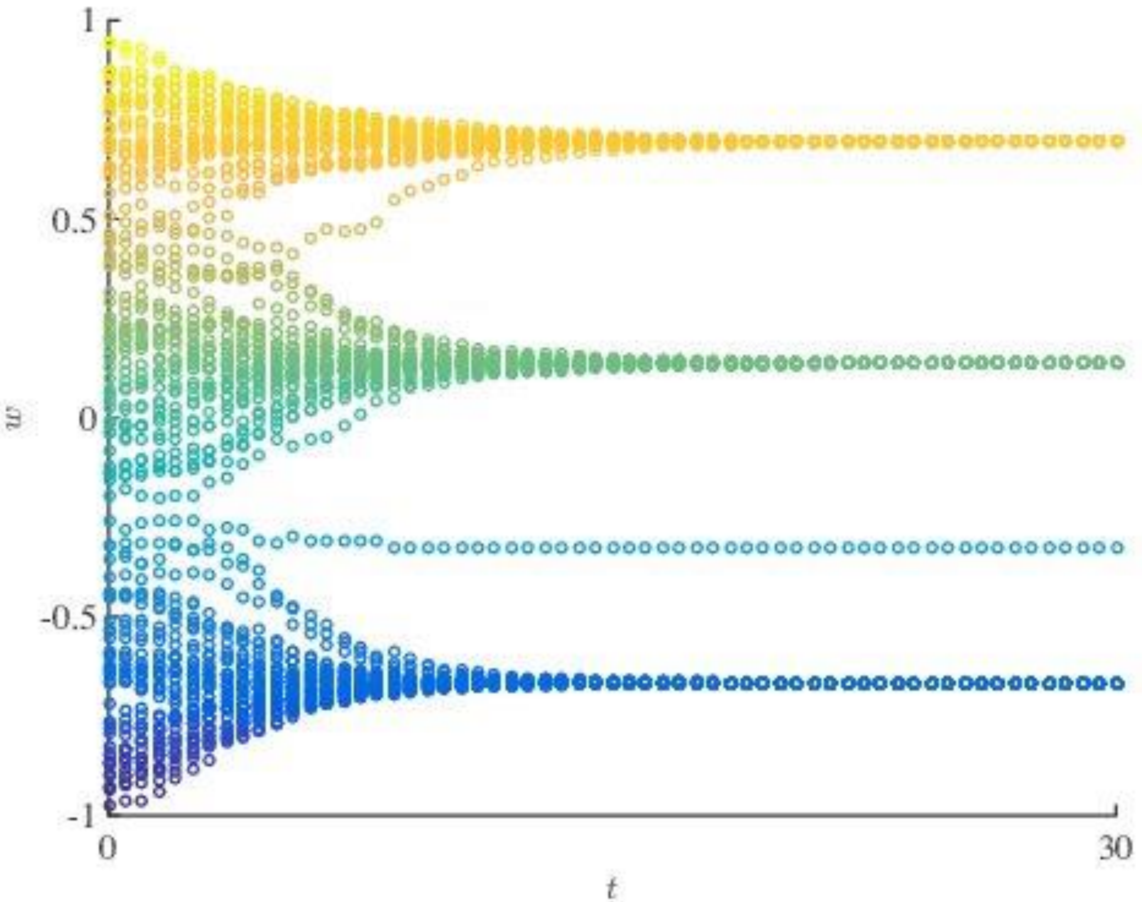
  - An agent *i* does not interact with *one* of its compatible neighbors (like in Deffuant), but with *all* its compatible neighbors at once.
  - intended to describe *formal meetings*

$$x_i(t+1) = \frac{\sum_{j:|x_i(t)-x_j(t)|<\epsilon} a_{ij} x_j(t)}{\sum_{j:|x_i(t)-x_j(t)|<\epsilon} a_{ij}}$$

- a_{ij}: elements of an adjacency matrix describing the communication network.
- Agent *i* takes the average opinion of its compatible neighbors.

Behavior of the Hegselmann-Krause model

- fully determined by the uncertainty ε
- Need lot of computation power (due to the average calculation)



The dynamics develops similarly to the Deffuant model:

- Leads to the same pattern of stationary states, with the number of final opinion clusters decreasing if ε increases.
- for $\varepsilon > \varepsilon_c$ (a threshold) there can be only one cluster

Cultural dynamics

Multidimensional vector model

- Mostly: opinion: scalar variable

„culture”: a vector of variables

The typical questions are similar:

- what are the microscopic mechanisms that drive the formation of cultural domains?
- What is the ultimate fate of diversity?
- Is it bound to persist or all differences eventually disappear in the long run?
- What is the role of the social network structure?

Axelrod model

- Axelrod, 1997
- Attracted lot of interest both from social scientists and physicists
  - Reason (soc. sci): inclusion of two fundamental mechanisms:
    - Social influence: the tendency of individuals to become more similar when they interact
    - Homophily: the tendency of alikes to attract each other, so that they interact more frequently
  - These two ingredients were generally expected to generate a self-reinforcing dynamics leading to a global convergence to a single culture.
  - But it turns out that the model predicts in some cases the persistence of diversity. (The importance of minimal models!)
  - From the viewpoint of stat. phys:
    - is a "vectorial" generalization of opinion dynamics models
    - gives rise to a very rich and nontrivial phenomenology, with some genuinely novel behavior

Axelrod's model

- **Individuals** :
  - are nodes on a network (or on the sites of a regular lattice – original version)
  - They are endowed with *F* integer variables ($\sigma_1, \ldots, \sigma_F$) (describing their "culture")

    The variables are the "*cultural features*"
- **Each σ_i** (feature) can assume *q* values: $\sigma_f = 0,1,\ldots,q-1$
  - q: number of possible traits (modeling the different "beliefs, attitudes and behavior" of individuals
- **An elementary step**:
  - an individual *i* and one of his neighbors *j* are selected
  - The overlap between them is computed:

$$\omega_{i,j} = \frac{1}{F} \sum_{f=1}^{F} \delta_{\sigma_f(i),\sigma_f(j)}$$

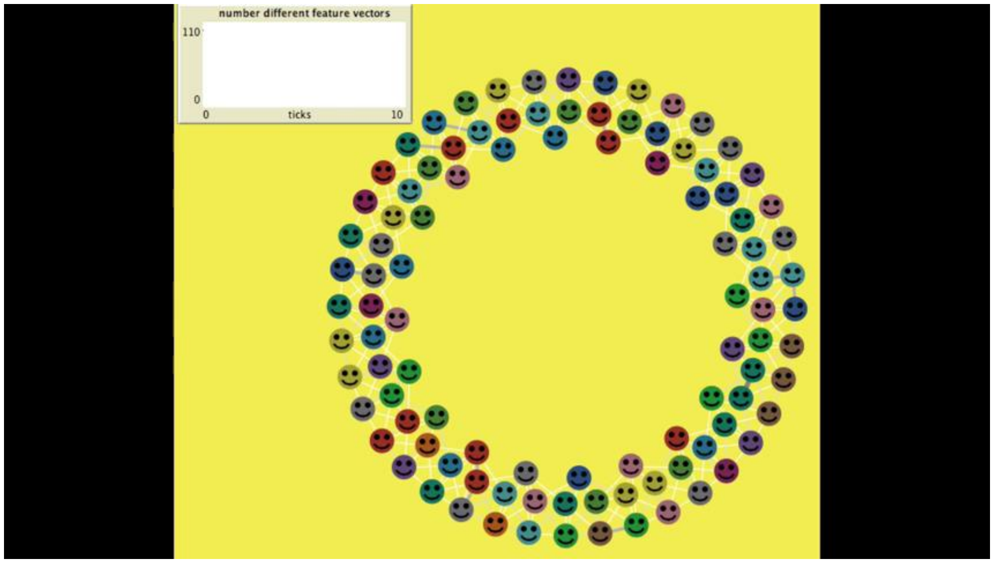
    Where $\delta_{i,j}$ is the Kronecker delta

  - $\omega_{i,j}$: probability of interaction between *i* and *j*
    - If they interact: one of the features for which traits are different ($\sigma_f(i) \neq \sigma_f(j)$) is selected and the trait of the neighbor is set equal to σ_f (i)
    - If they do not interact: nothing happens

Features of the Axelrod model

- the dynamics tends to make interacting individuals more similar
- Interaction:
  - more likely for neighbors already sharing many traits (homophily)
  - becomes impossible when no trait is the same
- For each pair of neighbors: two stable configurations:
  1. when they are exactly equal, so that they belong to the same cultural region or
  2. when they are completely different, i.e., they sit at the border between cultural regions
- Starting from a disordered initial condition:
  - The evolution on any finite system leads to one of the many **absorbing states**, which belong to **two classes**:
  1. the **ordered states**, in which **all** individuals have the same set of variables, or
  2. **Frozen states** with different coexisting cultural regions (more numerous)
- Which one is reached: depends on *q* (number of possible traits):
  - Small *q*: quickly full consensus is achieved
  - Large q: very few individuals share traits → few interactions occur → formation of small cultural domains that are not able to grow (disordered frozen state)

Axelrod's model of cultural dissemination in a circle network (16 sec)



Inset: number of different feature vectors

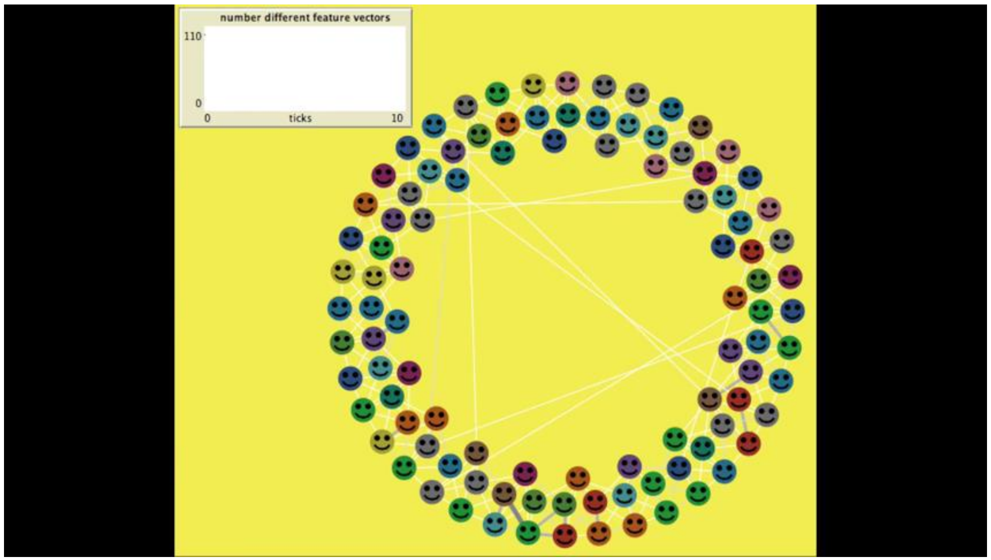
- a circle interaction structure
- 100 agents, each
  - with 6 network contacts
  - 5 features
- Each feature can adopt 15 values
- Init: random set of traits
- Color of agents: *one* of the features
- Color and thickness of the lines: the overall similarity between the respective nodes
  - black thick lines: identical traits on all features
  - White thin lines: the two nodes are connected but maximally different.

- emergence of internally homogenous but mutually different clusters.
- Dynamics settled after 34,809 iterations with 19 cultural clusters.

(Michael Maes, 2015)

https://www.youtube.com/watch?v=kLPIeQAKQQw

Axelrod's model of cultural dissemination in a small world network (47 sec)



- a small-world interaction structure
- 100 agents, each
  - with 6 network contacts
  - 5 features
- Each feature can adopt 15 values
- Init: random set of traits
- Color of agents: *one* of the features
- Color and thickness of the lines: the overall similarity between the respective nodes

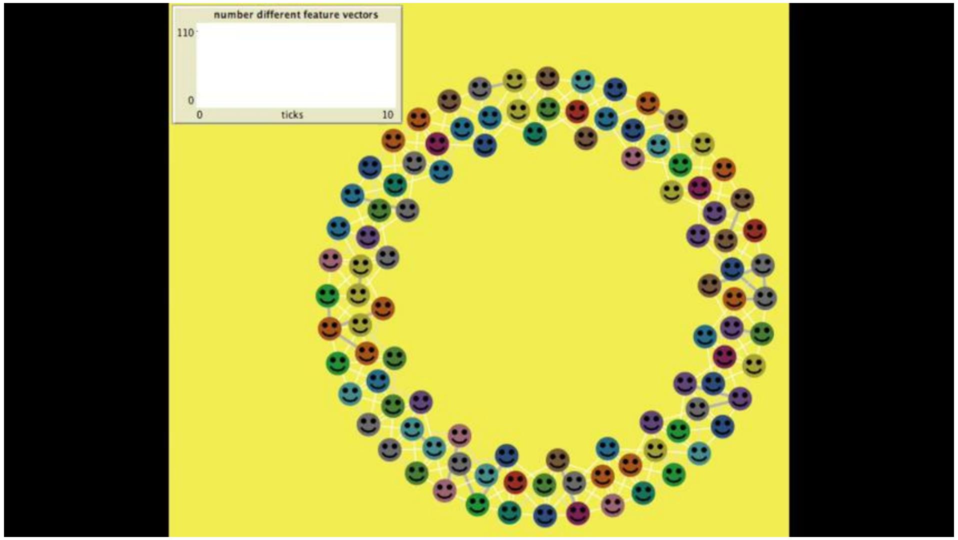
- emergence of internally homogenous but mutually maximally different clusters.
- Dynamics settled after 140,427 iterations with 7 cultural clusters.

Inset: number of different feature vectors

(Michael Maes, 2015)

https://www.youtube.com/watch?v=7ZbVUNWrLYs

Playing with Axelrod's model: the effect of globalization (53 sec)



- Globalization: more individuals are in contact with others who are geographically very distant
- Circle NW interaction structure (at the beginning!)
- 100 agents, each
  - with 6 network contacts
  - 5 features
- Each feature can adopt 15 values
- Init: random set of traits
- Color of agents: *one* of the features
- Color and thickness of the lines: the overall similarity between the respective nodes
- The dynamics reaches a rest point (after 51,065 iterations)
- Rewire 20 links and cont. (modeling that individuals have more contact to distant others)

  (Michael Maes, 2015)

https://www.youtube.com/watch?v=VvXjk8P4TX0

Illustrates two implications of the model:

1. due to the rewiring the number of clusters in equilibrium decreased from 22 to 16
2. after the simulation continued (after rewiring) the number of unique combinations of cultural traits (diversity) first increased and then decreased

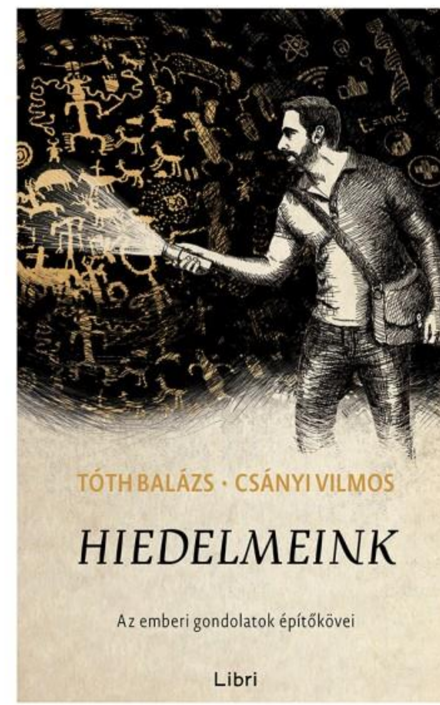
(i) globalization decreases cultural diversity

(ii) the short-term effects differ from the long-term effects

When the „elements of the belief system” (that is: „beliefs”) are interrelated

Most models assuming interrelated beliefs are coming from the political science

how people form their political attitudes

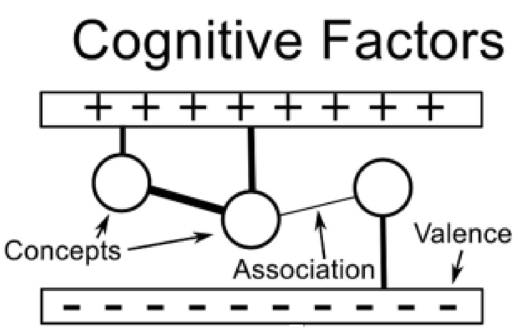


Two crucial aspects of belief dynamics

Cognitive bias (or belief bias):

- **Def**: A person's tendency to accept arguments that supports a conclusion that aligns with his/her values, beliefs and prior knowledge, while rejecting counter arguments to the conclusion
- Leads to individual belief rigidity

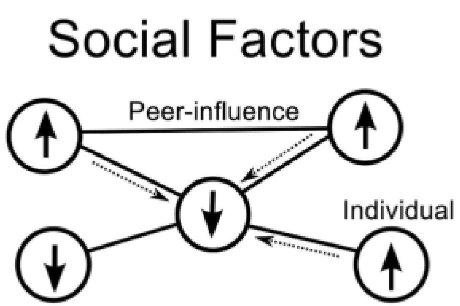
- Cognitive dissonance (well-studied area)



Social influence:

- The tendency of individuals to become more similar when they interact (we have seen it at the Axelrod model)

- Leads to social conformity

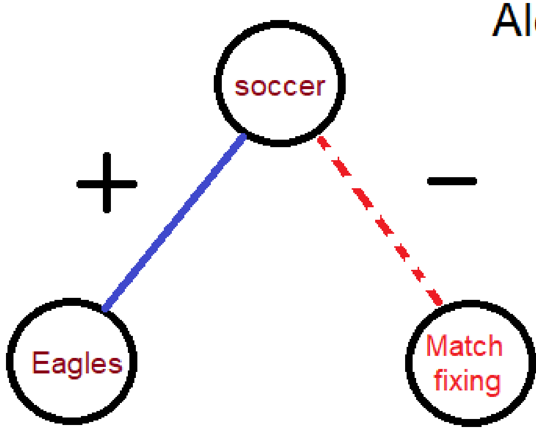


Rodriguez N, Bollen J, Ahn Y-Y (2016) Collective Dynamics of Belief Evolution under Cognitive Coherence and Social Conformity. PLoS ONE 11(11): e0165910. doi:10.1371/journal.pone.0165910

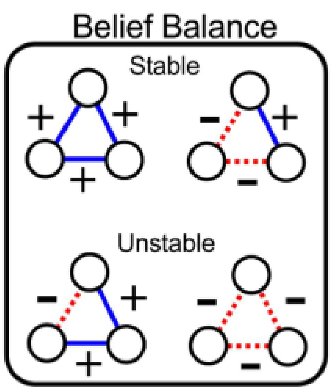
A cognitive-social model

- Individuals are embedded into a social NW, and social influence takes place via the social ties
- Each individual possesses a *network* of concepts and beliefs
- The internal (in)coherence of each individual's belief network is evaluated

The belief system of Alex, the soccer fanatic



1) Alex loves soccer
2) Alex supports the Eagles



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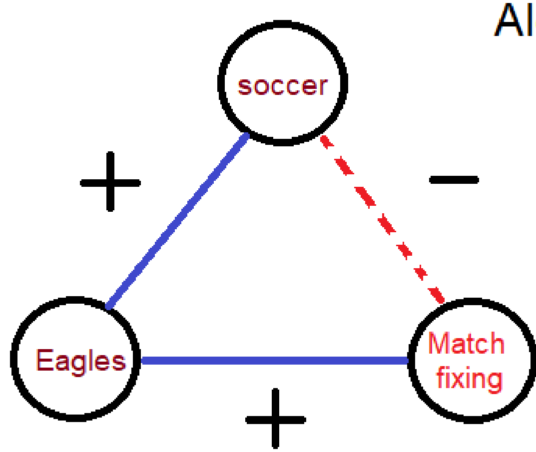
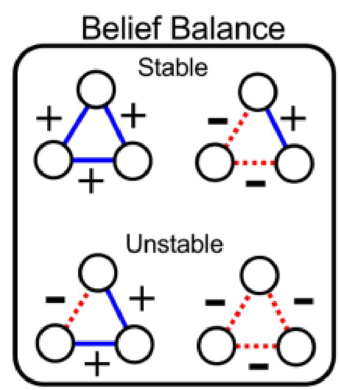


1) Alex loves soccer
2) Alex supports the Eagles
3) The eagles has been with match fixing



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A cognitive-social model

- The *internal coherence* of each individual's belief network is evaluated by the *internal energy function*

(on the belief NW *M*):

(For simplicity, the belief NW is complete, meaning that all concepts have a positive or negative association with every other)

$$E_n^{(i)} = -\frac{1}{\binom{M}{3}} \sum_{j,k,l} a_{jk} a_{kl} a_{jl}$$

- The evolution of belief systems is also driven by social interactions: *social energy* term, capturing the *degree of alignment* between connected individuals.)

$$E_n^{(s)} = -\frac{1}{k_{max}\binom{M}{2}} \sum_{q \in \Gamma(n)} \vec{S}_n \cdot \vec{S}_q$$

k_{max} is a normalization factor, maximum degree of N.

S: belief state vector: each element corresponds to an *edge*

- Total energy:

where

I: peer-influence ,

J: coherentism

$$H = \sum_{n \in \mathcal{N}} \left[J E_n^{(i)} + I E_n^{(s)} \right]$$

- The status of the entire society is characterized by

(i) the average internal coherence of the individuals $\langle E^{(i)} \rangle$, and

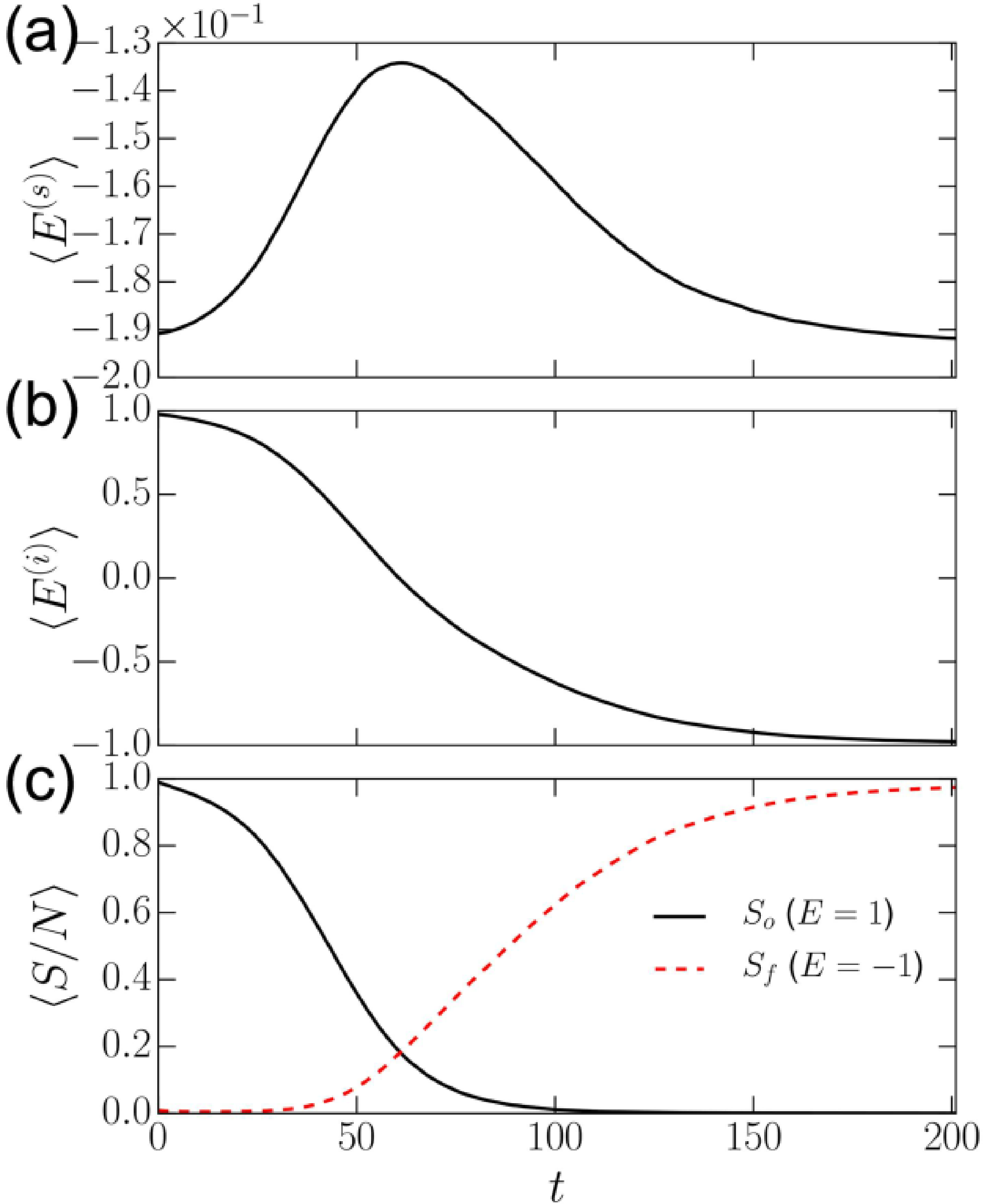
(ii) the homogeneity of the society $\langle E^{(s)} \rangle$

- The simulation:
  - At each time step a random pair of individuals is chosen
  - One of the individuals (sender)randomly chooses a belief (association) from its internal belief system and sends it to the other individual (receiver)
  - Assumption: each individual has an identical set of concept nodes
  - The receiver accepts it if it decreases its individual energy H_n
  - If $\Delta H_n > 0$, the receiver accepts it with probability $e^{\frac{-\Delta H_n}{T}}$
  - T is "susceptibility" / "open-mindedness"

Results

- Given a *homogeneous* population of people with *highly coherent* belief systems, society remains stable.
- Given a *homogeneous* population of *incoherent belief systems*, society will become unstable and following a small perturbation, breaks down

- In simulation:
  - The society is initialized at **consensus** with an **incoherent** belief system.
  - Then 1% of the population are given a random belief system
  - Individuals attempt to reduce the energy of their own belief systems and leave consensus



In the simulation, the society is initialized at consensus with an incoherent belief system. Then 1% of the population are given a random belief system.

Strong societal consensus does not guarantee a stable society in our model. If major paradigm shifts occur and make individual belief systems incoherent, then society may become unstable.

(a) The plot shows the evolution of social energy $E^{(s)}$ over time. The system starts at consensus but with incoherent beliefs. After introducing a small perturbation, individuals leave consensus, searching for more coherent sets of beliefs, until society reconverges at a stable configuration.

(b) Decreasing mean individual energies $<E_{(i)}>$ over time illustrates individual stabilization during societal transition.

(c) $<S/N>$ is the fractional group size. As society is upset, the original dominant but incoherent belief system S_o (solid black) is replaced by an emerging coherent alternative S_f (dashed red).