

# Hierarchy

## Part II

### Biologically Inspired Systems

Lecture 3

Nov 6, 2023

# A short reminder

- Last lecture we have seen:
  - Definition of hierarchy: control of behavior (characteristics)
  - Types of hierarchy:
    1. Order hierarchy
    2. Containment (Nested/Embedded hierarchy)
      - i. Subsumptive containment hierarchy (taxonomic)
      - ii. Compositional containment hierarchy (level h.)
    3. Flow (or control) hierarchy
  - Dominance hierarchy
  - Models for leadership (with preferred directions)

# Describing hierarchical structures

- Most commonly used mathematical tool: *graphs*
- Primarily they are connected to systems embodying *flow hierarchy*
  - observations, experiments, computer simulations are likely to return flow hierarchy;
  - all other hierarchy types can be transformed into flow hierarchy in a rather straightforward way
- We can measure the hierarchical level of the *graph* (not the system itself)
- No “most appropriate” measure (many structure is “matter of intuition / taste”)
- Most of the proposed measures take values on the  $[0, 1]$  interval
- Measures (circles, Random walk, GRC)

# Models for leadership

- Extension of the “Couzin model”
- No individual recognition, no signaling mechanism
- Non-informed individuals: are not required to know how many and which individuals has information
- Vice versa: Informed individuals are not required to know anything about the information-level of their mates and that how the quality of their information was compared to that of others.

## The model:

- **Rule 1:** highest priority
  - Individuals attempt to maintain a certain distance among themselves by turning away from those neighbors  $j$  which are within a certain distance towards the opposite direction:

$$\vec{d}_i(t + \Delta t) = - \sum_{j \neq i} \frac{\vec{r}_j(t) - \vec{r}_i(t)}{|\vec{r}_j(t) - \vec{r}_i(t)|}$$

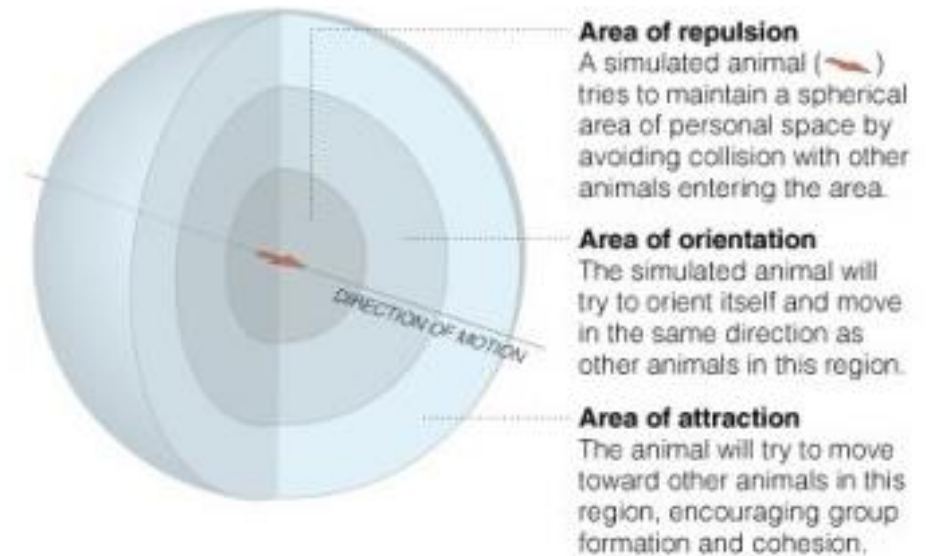
$\vec{d}_i$ : desired direction of individual  $i$

$\vec{r}_i$ : position of particle  $i$

$\vec{v}_i$ : direction of unit  $i$

## Simulating Swarm Intelligence

Researchers created a model of swarm behavior by programming individuals to maintain personal space while turning and moving in the same direction as others.



Sources: Iain D. Couzin, *Journal of Theoretical Biology*

# Models for leadership

## The model (cont):

- Rule 2

If there are no mates within the range of repulsion, than the individual will attempt to align with those neighbors  $j$ , which are within the range of alignment:

→ The desired direction:

$$\vec{d}_i(t + \Delta t) = - \sum_{j \neq i} \frac{\vec{r}_j(t) - \vec{r}_i(t)}{|\vec{r}_j(t) - \vec{r}_i(t)|} + \sum_{j \neq i} \frac{\vec{v}_j(t)}{|\vec{v}_j(t)|}$$

$\vec{d}_i$ : desired direction of individual  $i$

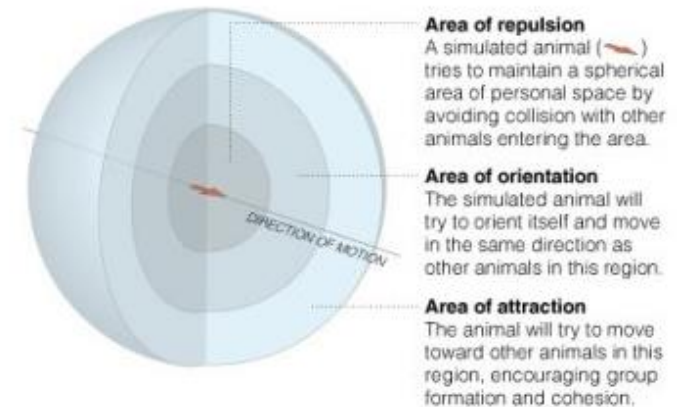
$\vec{r}_i$ : position of particle  $i$

$\vec{v}_i$ : direction of unit  $i$

- Corresponding unit vector:  $\hat{d}_i(t) = \vec{d}_i(t) / |\vec{d}_i(t)|$
- Introducing “influence”: a portion of the group ( $p$ ) is given information/motivation about a preferred direction, described by the (unit) vector  $\vec{g}$ .
- The rest of the group does not have directional preference.

### Simulating Swarm Intelligence

Researchers created a model of swarm behavior by programming individuals to maintain personal space while turning and moving in the same direction as others.



Sources: Iain D. Couzin, *Journal of Theoretical Biology*

Informed individuals balance their

- social alignment  $\hat{d}_i(t)$  (the unit vector of  $\vec{d}_i(t + \Delta t) = -\sum_{j \neq i} \frac{\vec{r}_j(t) - \vec{r}_i(t)}{|\vec{r}_j(t) - \vec{r}_i(t)|} + \sum_{j \neq i} \frac{\vec{v}_j(t)}{|\vec{v}_j(t)|}$ ) and
- preferred direction  $\vec{g}_i$

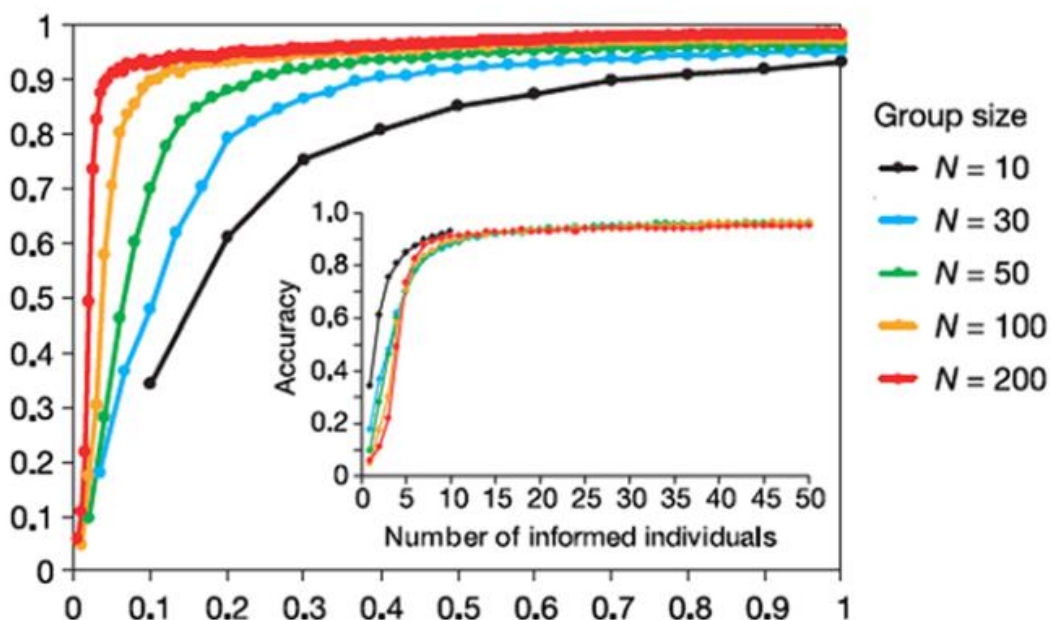
with the weighting factor  $\omega$ :

$$\vec{d}_i(t + \Delta t) = \frac{\hat{d}_i(t + \Delta t) + \omega \vec{g}_i}{|\hat{d}_i(t + \Delta t) + \omega \vec{g}_i|}$$

- $\omega$  can exceed 1: the individual is influenced more by its own preferences than by its mates
- “Accuracy” of the group: normalized angular deviation of the group direction around the preferred direction  $\vec{g}_i$

# Results:

- for fixed group size, the accuracy increases asymptotically as the portion  $p$  of the informed members increases (...that is...)
- the larger the group, the smaller the portion of informed members is needed, in order to guide the group towards a preferred direction



# Lessons

- Leadership might emerge from the differences of the level of information possessed by the group members
- information can be pertinent → leadership can be transient and transferable too

# Experiments with homing pigeons

- **10 homing pigeons** flying in flocks
- high-precision lightweight GPS
- Two kind of flights were recorded:
  1. spontaneous flights near the home loft (“**free flights**”) and
  2. during **homing** following displacement to distances of approximately **15 km** from the loft (“homing flights”)



Trajectories of a flock of nine pigeons during a homing flight



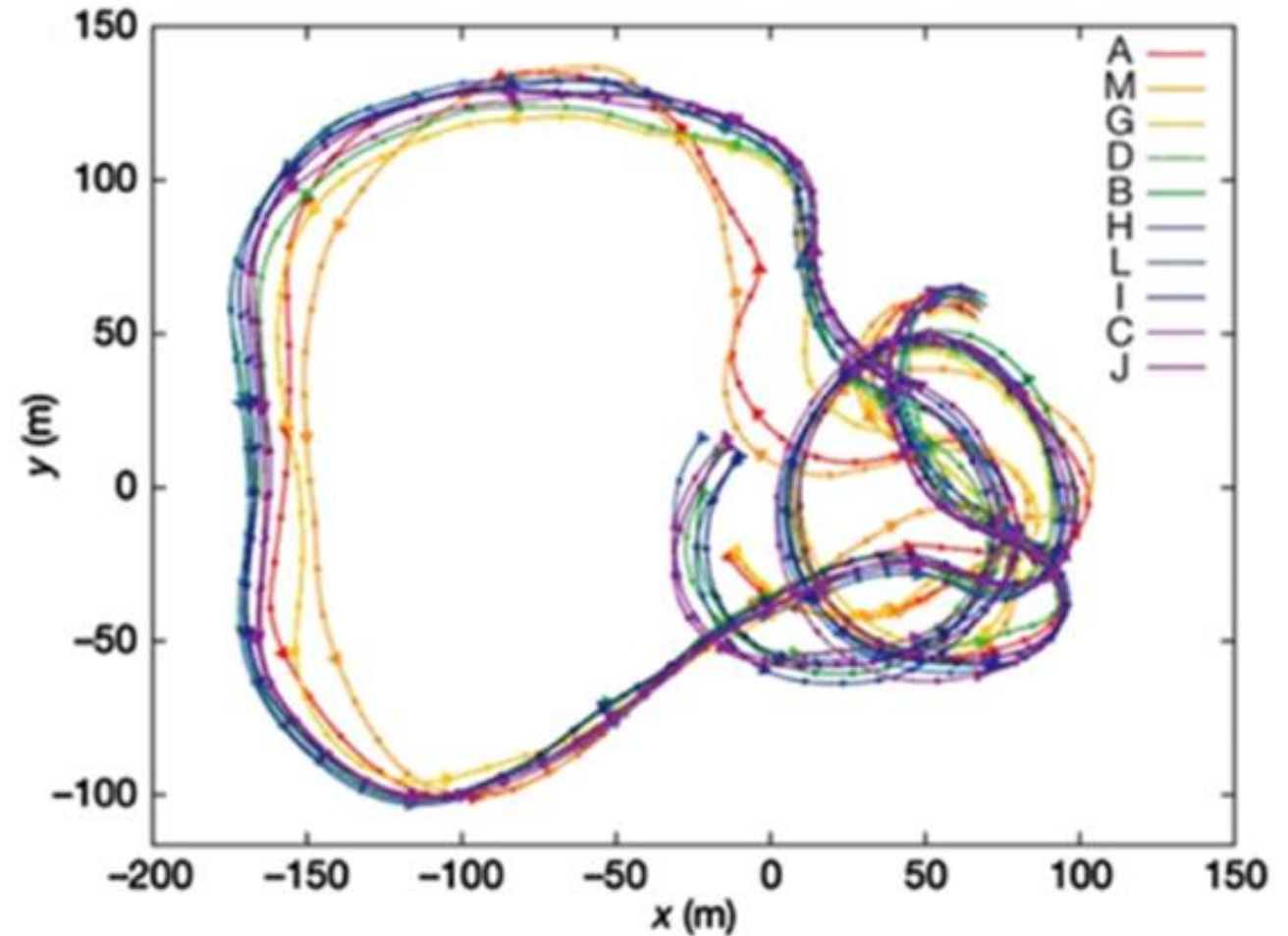
# Analysis

- **Goal:** to find out how homing pigeons navigate collectively (leadership hierarchy)
  - The *influence* of the birds' behavior on its fellow flock members and on the flock
- → **temporal relationship** between the bird's flight direction and those of others
- “**Leading event**”: when a bird's direction of motion was “copied” by another bird, delayed in time.

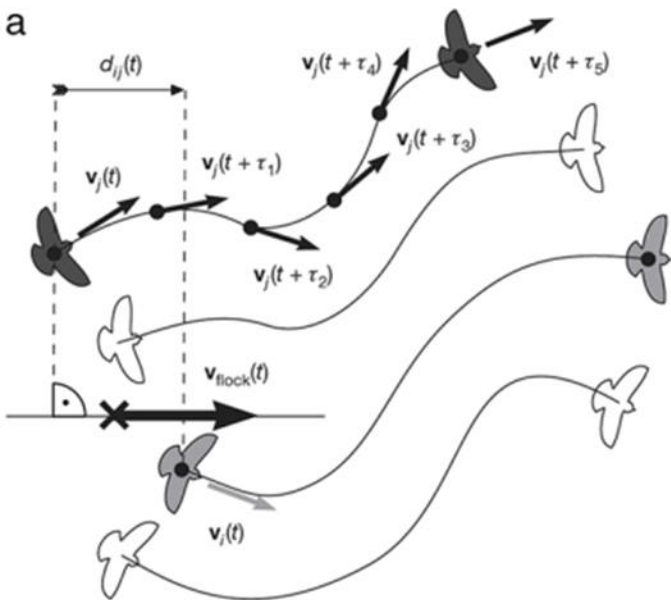
This was quantified by determining the **directional correlation delay time** ( $\tau^*_{ij}$ ) (measured in seconds) from the maximum value of the **directional correlation function**

$$C_{ij}(\tau) = \langle \vec{v}_i(t) \cdot \vec{v}_j(t + \tau) \rangle$$

brackets: time average for each pair of birds  $i, j$



2-minute segment from a free flight performed by a flock of ten pigeons in the vicinity of the loft. The smaller and the larger dots indicate every 1s and 5s, respectively. Each path begins near the center of the plot. Letters refer to bird identity.



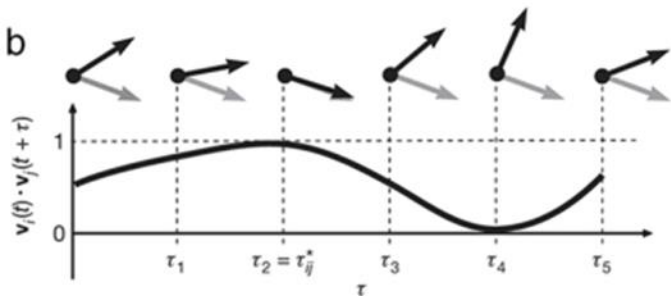
# Yielding the directional correlation function

a

- light grey: bird  $i$
- dark grey: bird  $j$
- For each pair ( $i \neq j$ ) the directional correlation function is

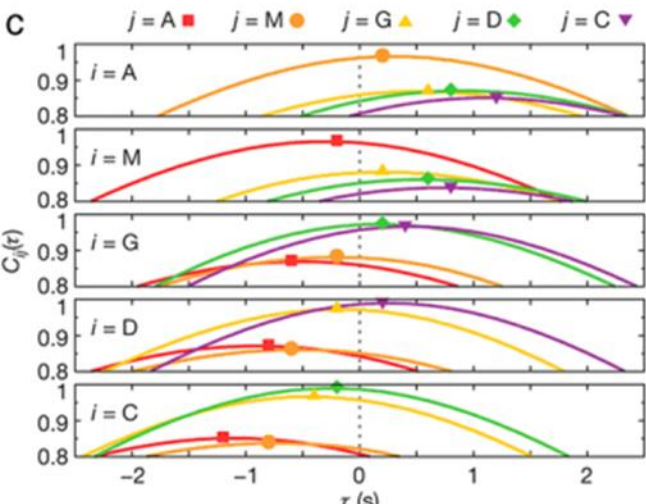
$$C_{ij}(\tau) = \langle \vec{v}_i(t) \cdot \vec{v}_j(t + \tau) \rangle$$

- The arrows show the direction of motion,  $\vec{v}_i(t)$



b

- Visualization of scalar product of the normalized velocity of bird  $i$  at time  $t$  and that of bird  $j$  at time  $t + \tau$ . In this example bird  $j$  is following bird  $i$  with correlation time  $\tau_{ij}^*$ .

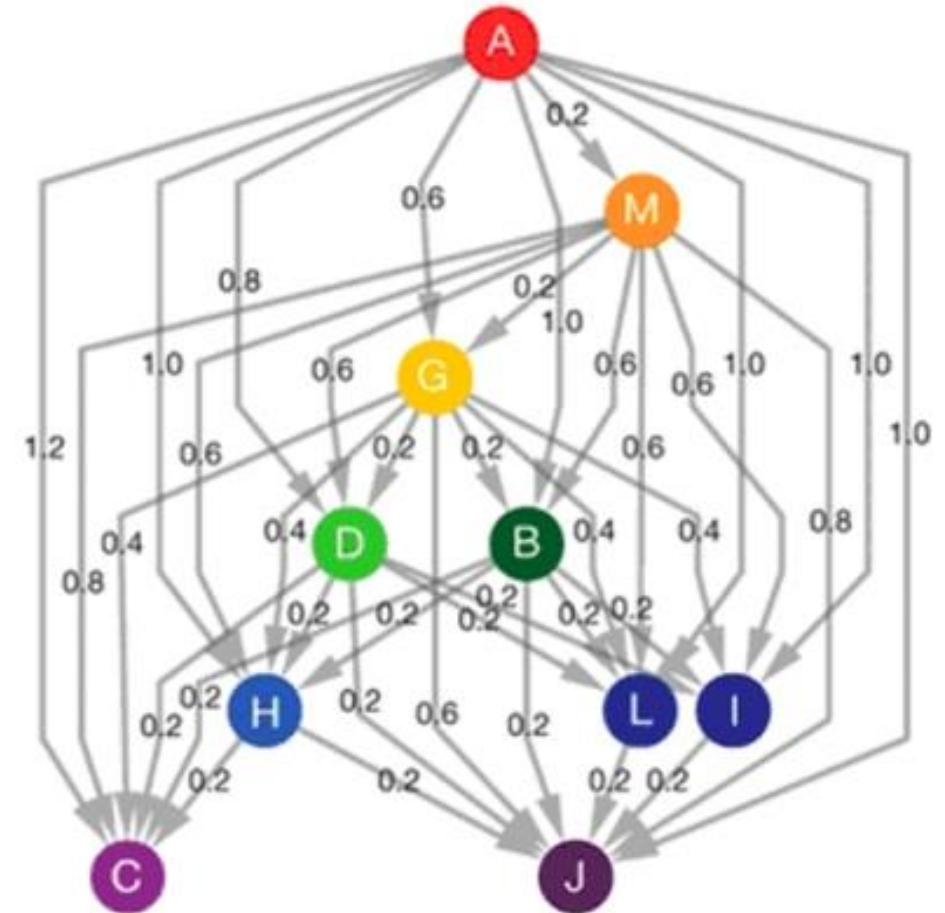


c

- The directional correlation function  $C_{ij}(\tau)$  during the flock flight. For more transparency only the data of birds A, M, G, D and C (in the order of hierarchy for that flight) are shown. The solid symbols indicate the maximum value of the correlation function,  $\tau_{ij}^*$ .
- These  $\tau_{ij}^*$  values were used to compose the directional leader-follower networks.

# Hierarchical leadership network generated for a single flock flight

- The directed edge points from the leader to the follower (i.e., the average directional correlation delay time for that pair,  $\overline{\tau_{ij}}$ , is positive);
- Values on edges show the time delay (in seconds) in the two birds' motion.
- For pairs of birds not connected by edges directionality could not be resolved at  $C_{min} = 0.5$ .





# Leadership vs. dominance - a systematic study

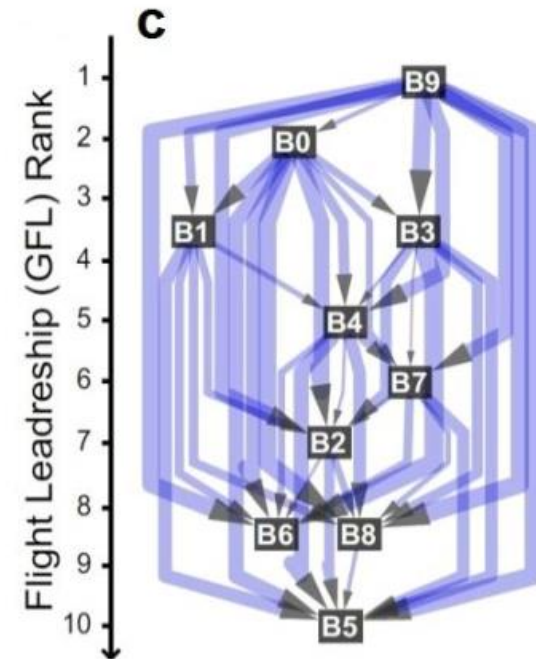
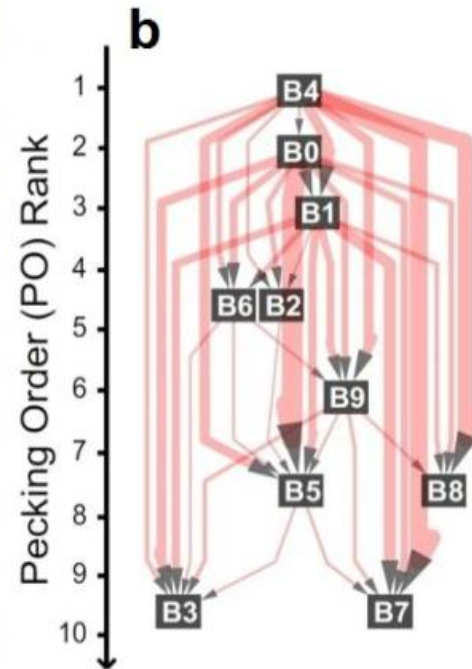
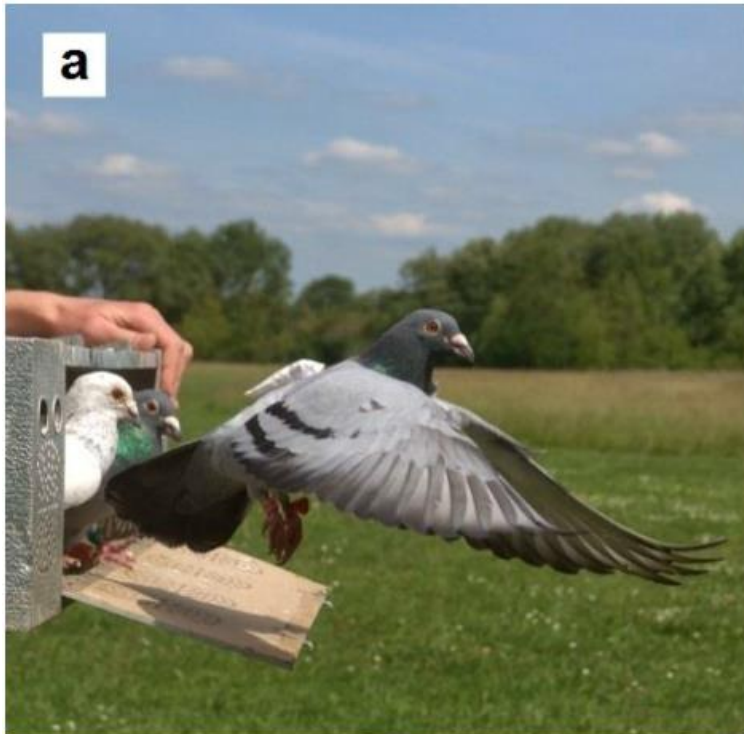
## *Do dominant individuals lead?*

- Flock of 10 pigeons
- L-F hierarchy was determined based on the directional correlation function analysis
- Dominance hierarchy was also determined (in the same group), based on computer-vision methods
- The first automated analysis of dominance relationships
- Both structure is clearly hierarchical



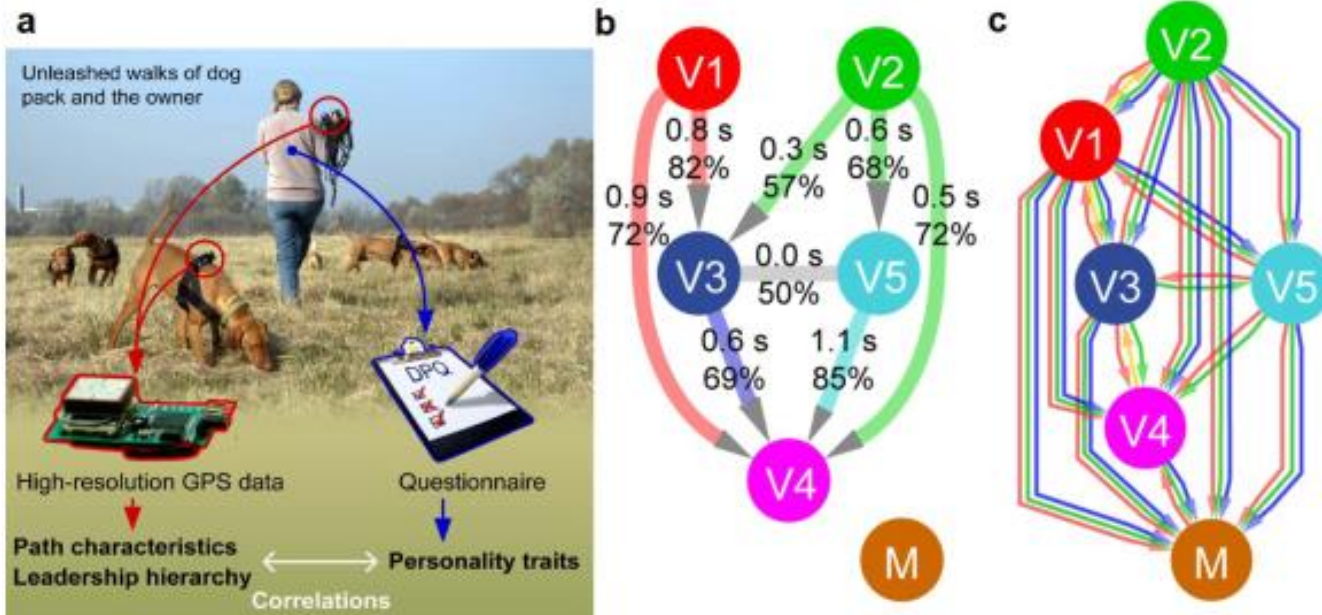
# Leadership vs. dominance – Results

- Dominance and leadership hierarchies are independent of each other!
- They can coexist within the same group without any kind of conflict: when it comes to collective travel those will lead the group who have better navigation skills (or information, etc.) and when it comes to feeding, mating, etc., dominance will decide.
- Hierarchy is context-dependent!





# Dominance vs. leadership hierarchy in dogs



- 6 dogs, belonging to the same household
- GPS logs during more than a dozen 30- to 40-minute unleashed walks, accompanied by their owner
- All the dogs were “Vizsla”, except for the one marked with “M”, which was a mixed-breed. This dog did not participate in the vizsla-network.

## b) Leader-follower hierarchy

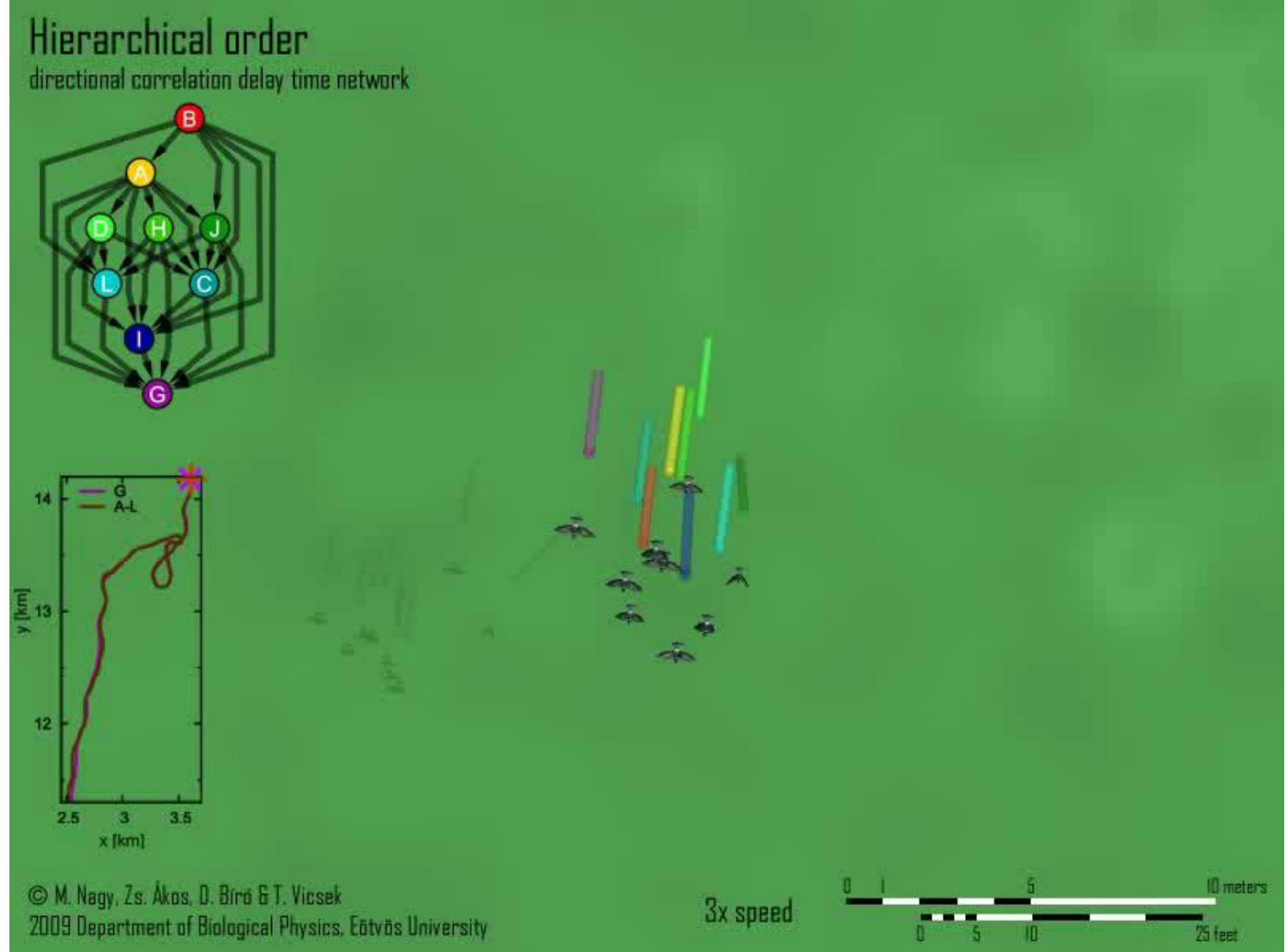
- The basis of creating the L-F NW was the directional delay time analysis
- The directed links: point from the leader towards the follower.
- Characteristic delay times are shown on the arrows (upper values).
- Lower values indicate the portion that the leader of that pair was actually leading.

## c) Dominance network of the dogs

- derived from a questionnaire.
- The arrows point from the dominant individual towards the subordinate.
- The colors represent the context of the dominance:
  - red: barking,
  - orange: licking the mouth,
  - green: eating
  - blue: fighting.

# “How much” knowledge is enough?

- *high resolution GPS data*
- *hierarchy of their leading-following behavior*
- Why do an individual follow an other?
- The ones that are being followed are simply more self-willed or they are better informed?
- How accurate knowledge is needed to reach the target? Etc.



**Hierarchical group dynamics in pigeon flocks,**  
M. Nagy et al. *Nature* **464**, 890-893 2010

# Formulating the problem:

- Given a flock of boids and a pre-defined target
- The flock has to reach the target (together) in the shortest possible way
- The units interact with each other
- The average knowledge is restricted

**Question:** how to distribute the available amount of knowledge among the group members in order to achieve the best group-performance?





# New direction depends on:

1. The average direction of neighbors (units within the “Range of Interaction, ROI”)  $\langle \vartheta_j^t \rangle_R$
2. Own estimation  $\theta_i^t + \eta_i^t$
3. Noise  $\xi_i^t$

(Discrete time, constant speed magnitude)

$$\vartheta_i^{t+1} = (1 - \lambda_i)(\theta_i^{t+1} + \eta_i^{t+1}) \oplus \lambda_i \langle \vartheta_j^t \rangle_R \oplus \xi_i^{t+1}$$

$\lambda_i$  : a parameter expressing how disposed boid  $i$  is to follow others. “Pliancy”

$\vartheta_i^t$  : the direction of boid  $i$  at time-step  $t$

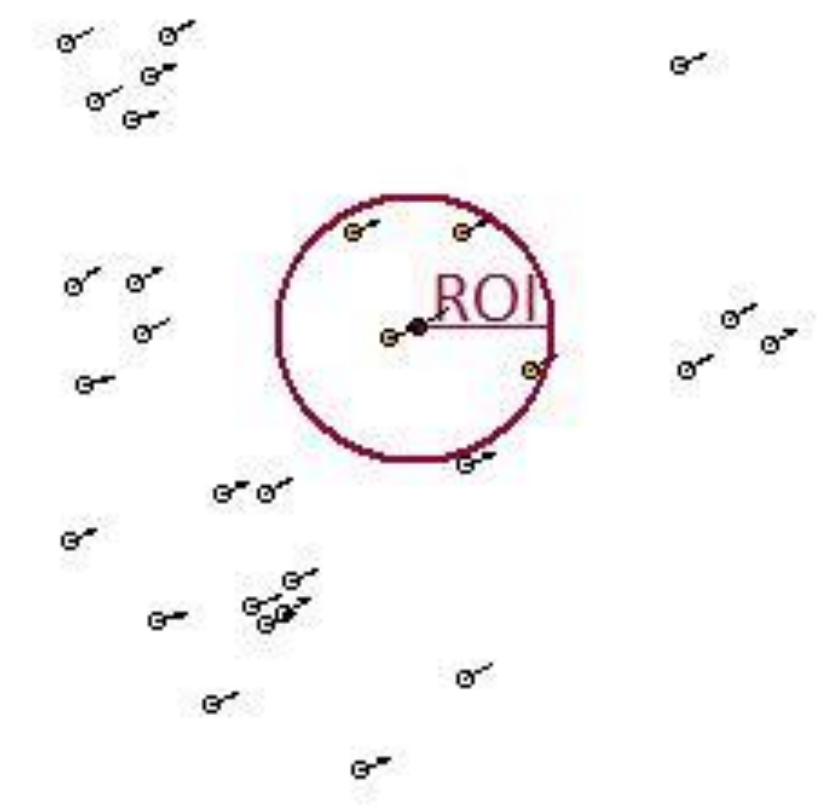
$\theta_i^t$  : the proper direction from boid  $i$  towards the target at time-step  $t$

$\eta_i^t$  : the actual estimation error of boid  $i$  at time-step  $t$

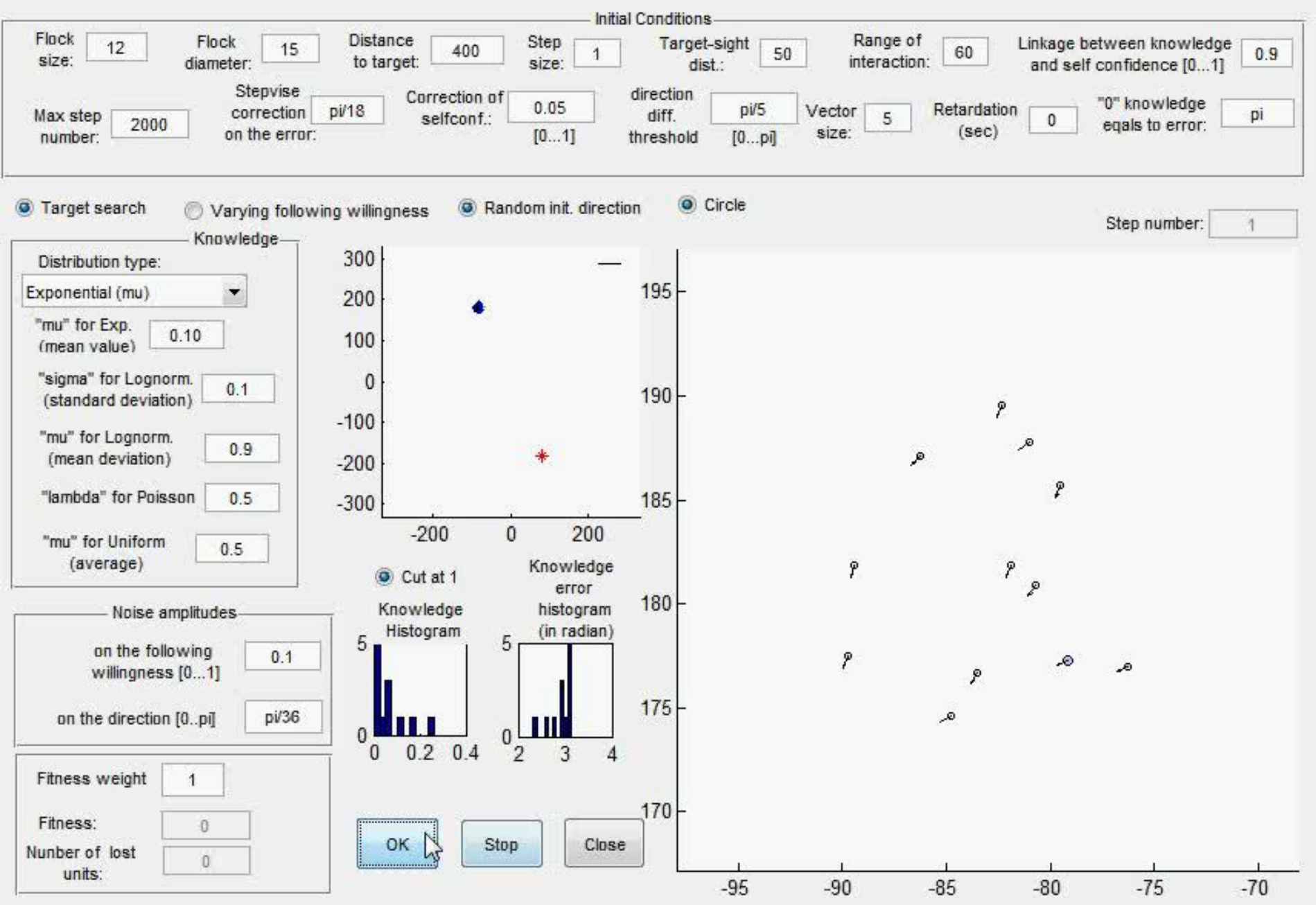
$\xi_i^t$  : random noise.  $|\xi_i^t| \leq \Xi$  where  $\Xi$  is the noise amplitude.

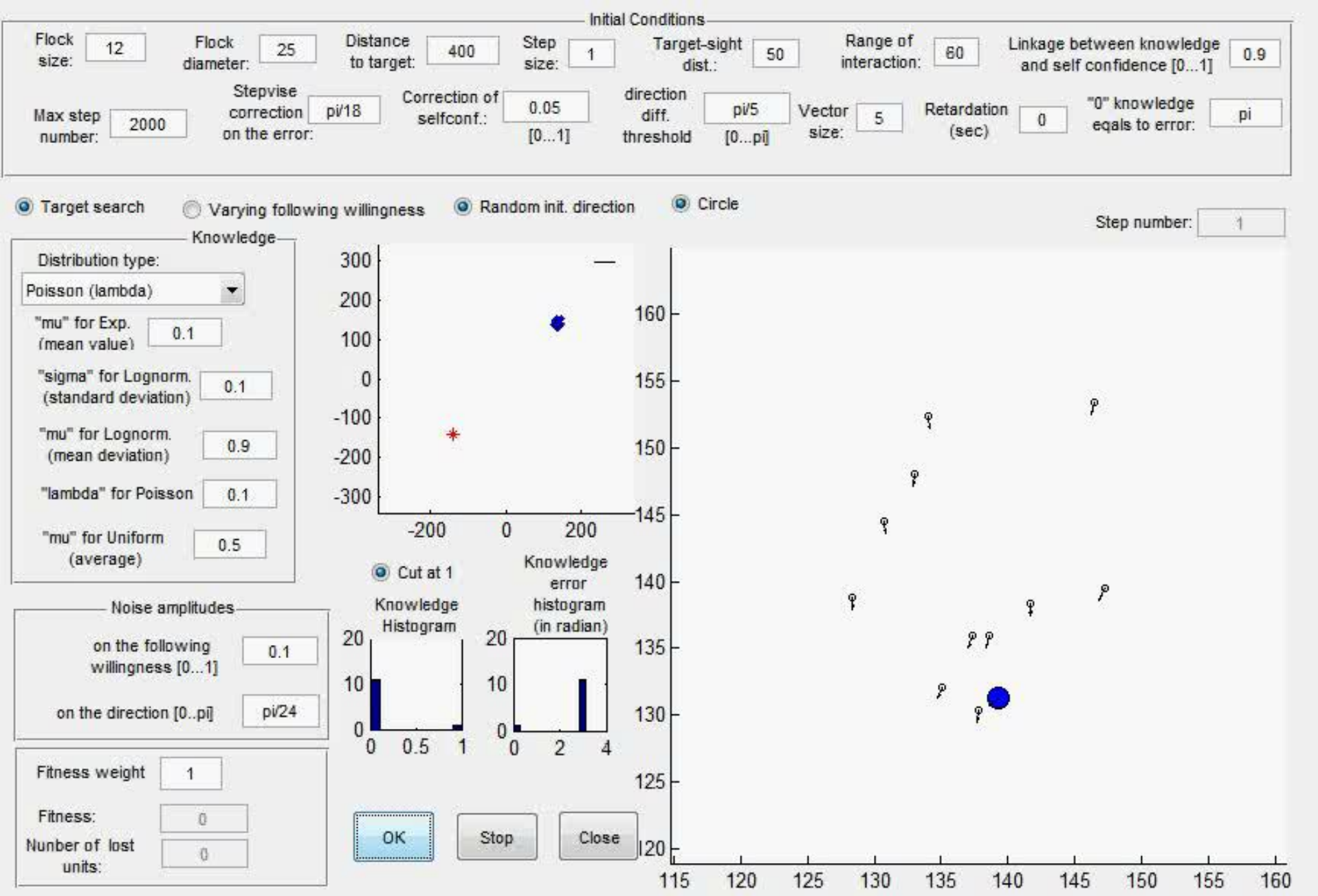
$\oplus$  : direction-summation

$\langle \vartheta_j^t \rangle_R$  : the average direction of the boids  $j$  being within the range of interaction,  $R$  of boid  $i$  at time-step  $t$



Flock size = 12,  
Exponential  
knowledge  
distribution,  
 $\mu=0.1$ ,  
coded in  
MatLab.





Flock size = 12,  
 "Two-valued"  
 knowledge  
 distribution,  
 $\mu=0.1$ ,  
 coded in MatLab.

Initial Conditions

Flock size:  Flock diameter:  Distance to target:  Step size:  Target-sight dist.:  Range of interaction:  Linkage between knowledge and self confidence [0...1]:

Max step number:  Stepwise correction on the error:  Correction of selfconf.:  [0...1] direction diff. threshold:  [0...pi] Vector size:  Retardation (sec):  "0" knowledge equals to error:

☒ Target search
 ☐ Varying following willingness
 ☒ Random init. direction
 ☒ Circle

Step number:

Knowledge

Distribution type:

"mu" for Exp. (mean value):

"sigma" for Lognorm. (standard deviation):

"mu" for Lognorm. (mean deviation):

"lambda" for Poisson:

"mu" for Uniform (average):

Noise amplitudes

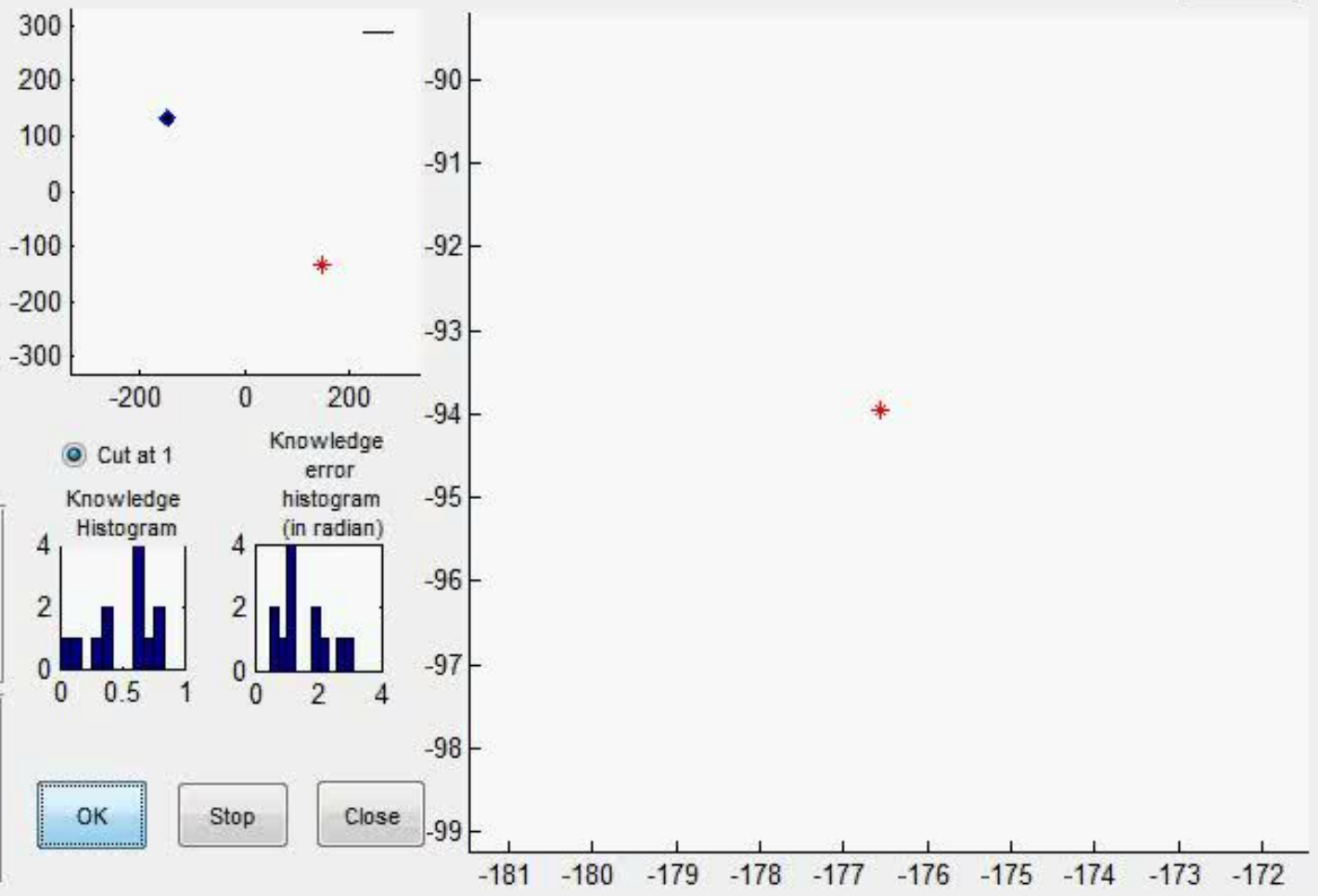
on the following willingness [0...1]:

on the direction [0..pi]:

Fitness weight:

Fitness:

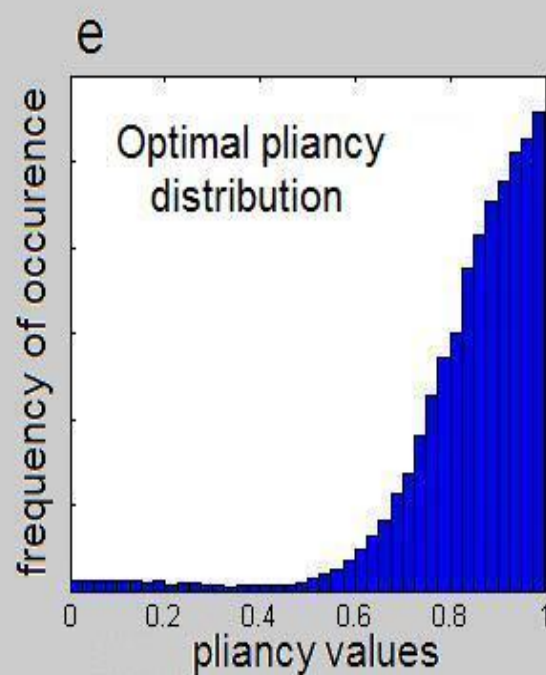
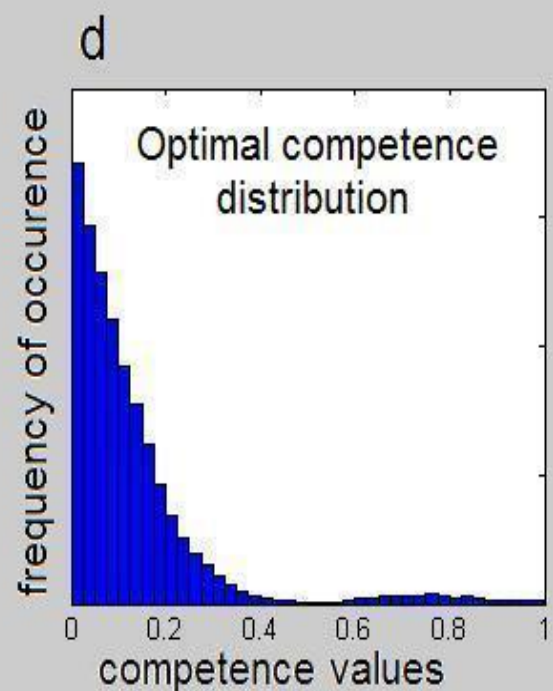
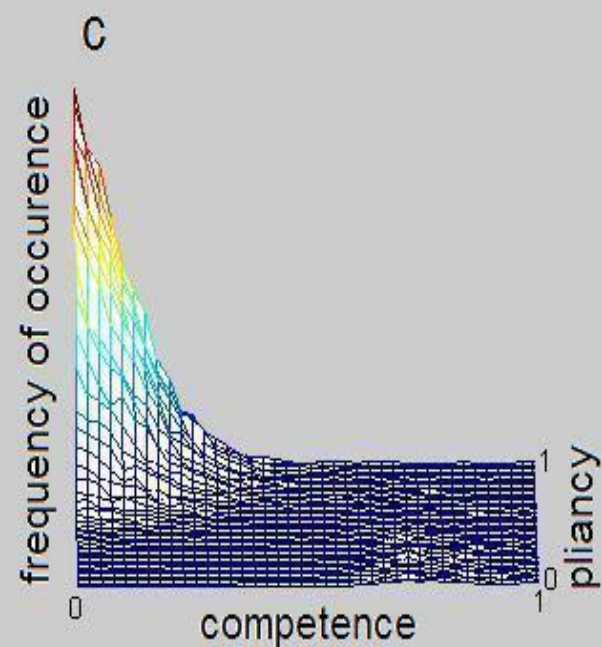
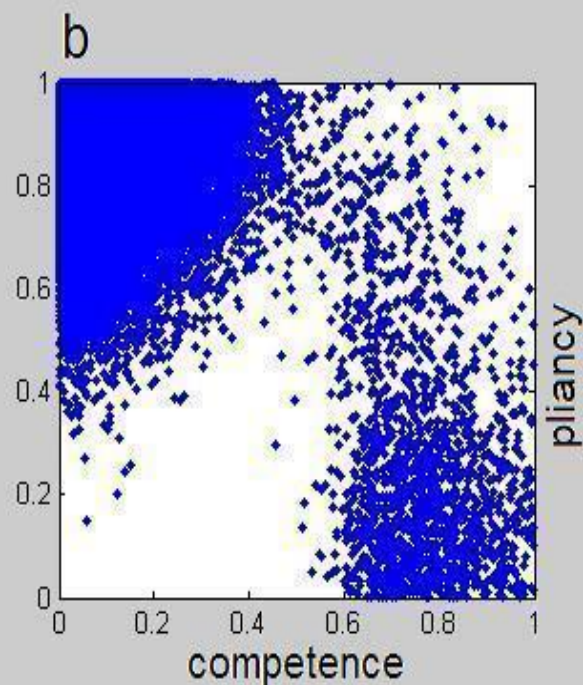
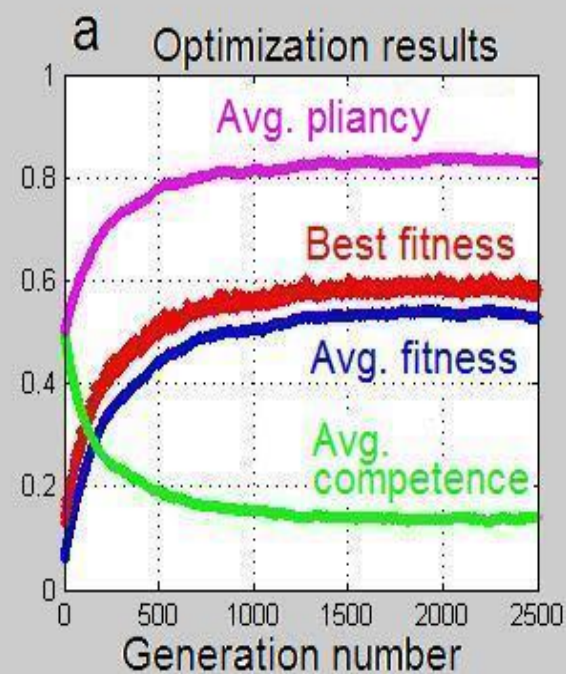
Number of lost units:



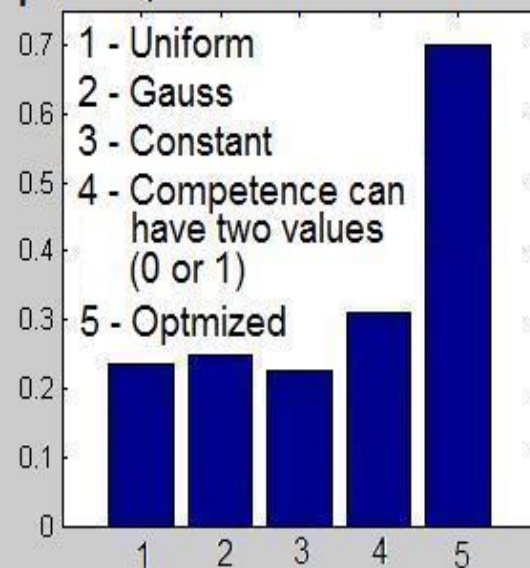
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Uniform  
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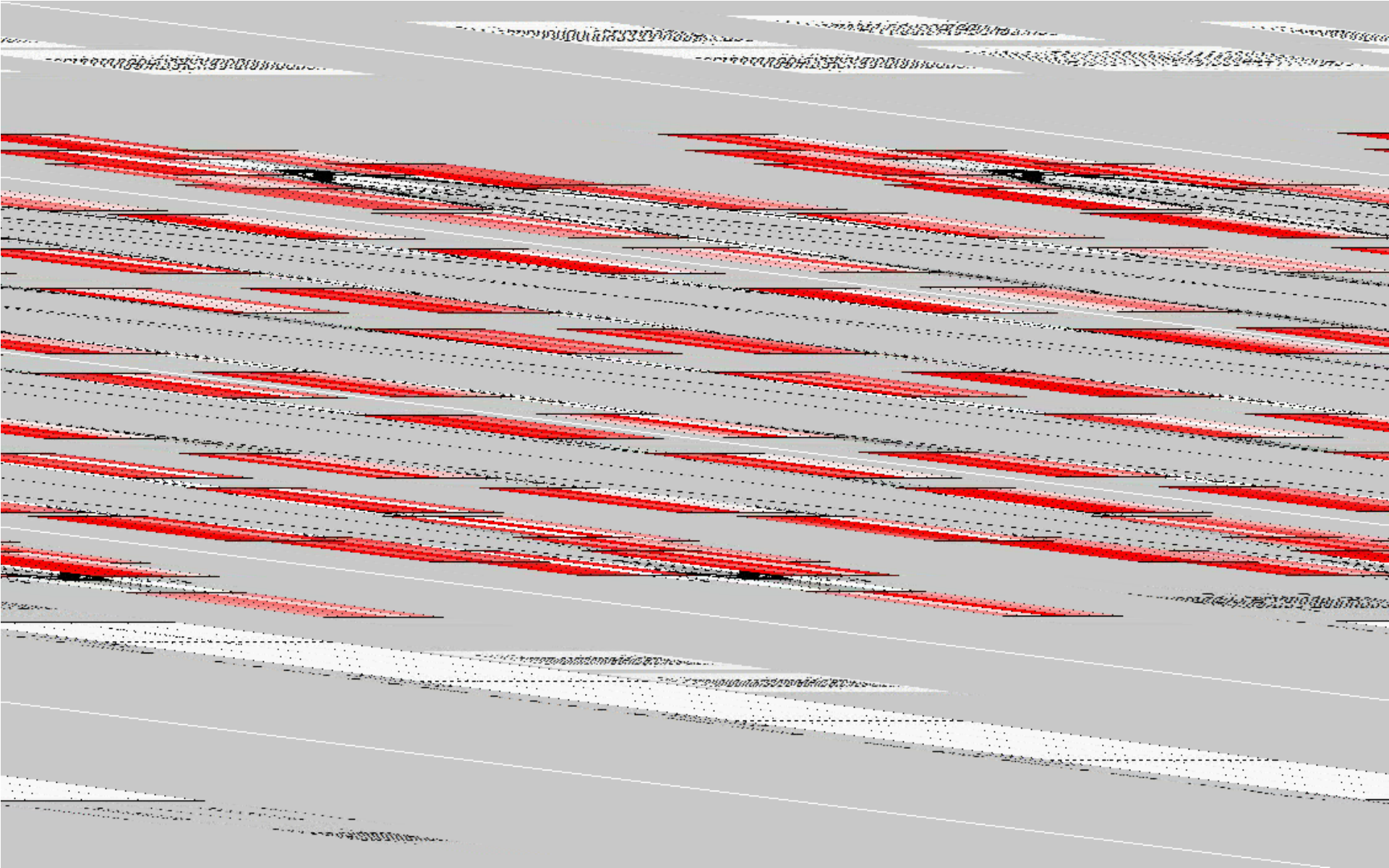
Group performance for various competence distributions



## Conclusions of the simulations:

- The *average* knowledge level can be surprisingly small
  - the individual estimations are very imprecise,
  - the knowledge value of most boids can be zero or near-to zero
- The way knowledge is distributed has a huge effect
- It helps, if
  - the units pay attention for their neighbors' movement
  - the pliancy and the knowledge values are inversely related

# Sequence guessing game on a Small-World NW

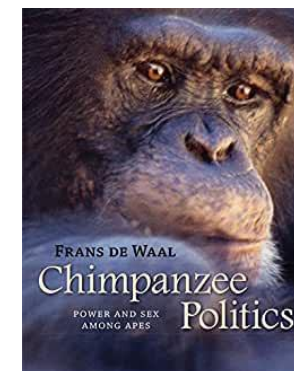


# Hierarchy in humans



# Dominance hierarchy

- A mechanism is needed to reduce the level of aggression triggered by the competition
- Regulate access to resources.
- The mechanism is simple: higher ranked individuals have primacy compared to their lower level mates.
- As one advances in the evolutionary tree, the structure of the dominance hierarchy gets more and more pronounced and complex, accompanied by more and more sophisticated strategies by which individuals try to get higher and higher ranks.
- Chimpanzees (few decades ago believed to be solely human):
  - coalition formation
  - manipulation
  - exchange of social favors
  - adaptation of rational strategies
- Obvious advantage: less fight

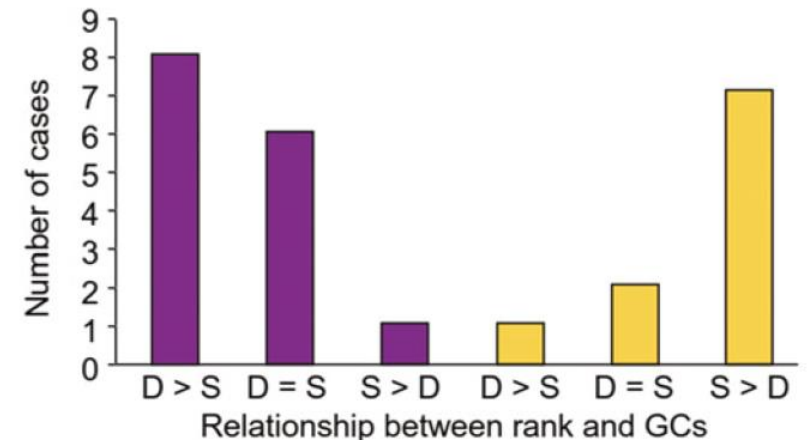


# Dominance hierarchy in humans

- Pretty much is known about the way it works in the animal world.
- Well-defined hormones and brain structures
- From a physiological point of view: the mechanisms determining the rank of an individual are very similar between mammals (incl. primates and humans)
  - **Testosterone**: (the principal male sex hormone)
    - level in the blood indicates the rank
    - In humans as well:
      - Experiments: tennis players, medical students
    - The level of the testosterone hormone and the inclination towards behaving dominantly form a positive feedback loop, as one intensifies the other.

# Dominance hierarchy in humans

- **Glucocorticoid steroid hormone (“stress hormone”):**
  - Not entirely clear picture (contradictory findings)
  - original view: subordinate individuals must be exposed to a much higher level of stress
  - Some measurements revealed the opposite
  - Some other: glucocorticoid secretion is stronger in lower-ranking individuals in general, from which the only exception is the alpha male at the very top, whose cortisol level is the highest in the whole group
  - the correlation between the level of stress hormone and high rank was found to be the strongest during periods of social instability
  - The observed differences might be due to the variations in the social organizations of different species and populations
  - in species, in which cooperative breeding is common, rank and stress hormone level are in direct proportion
  - in other species, they are in inverse proportion (this is one explanation)



# Human vs. non-human groups

- Groups of animals are **genetically open**, and over a period of time, a significant part of the group migrates or changes, so that **group identity independent of individuals does not develop**.
- anatomically modern humans appeared ~200,000 years ago
- ~120,000 – 50,000 years ago: “cognitive revolution”
- Human groups are characterized by
  - a decline in individual competition and
  - an increase in competition between groups (Bohem 1997).
- An **autonomous group entity**, independent of individuals and individuals, emerges as a new level of social organization, whose own interests are able to overshadow the individual interests of group members. (*history begins*)
- People living in human-like groups are characterized by the following genetically based species-specific traits:
  - 1. They like to participate in joint actions,
  - 2. They like to make common constructs (abstract, material, social),
  - 3. They form and accept common beliefs,
  - 4. They are loyal to their group and the group in order to push their own interests into the background, they even sacrifice their lives for the group.

Abstract way of thinking and more tightly connected groups go hand in hand (positive feedback loop)

# Hierarchy in humans

- ~120,000 – 50,000 years ago: “cognitive revolution”
  - new way of social self-organization:
    - formal roles (chieftain, king, pharaoh, colonel, etc.)
      - social ranks are independent of the actual individuals occupying them
      - positions can be organized into any kind of hierarchical system (including egalitarianism)
    - creating and following social rules
      - Support and reinforce the social structure
- Transition from small scale communities to large-scale societies?

Dominance hierarchy	Cultural hierarchy
<ul style="list-style-type: none"><li>• Genetically coded<ul style="list-style-type: none"><li>→ Restricted variability: the basic features are the same within one species.</li></ul></li><li>• Controlled mainly by hormones (testosterone, stress hormones, etc.)<ul style="list-style-type: none"><li>→ Mostly instinctive</li></ul></li><li>• Its main purpose is to minimise the inner-group aggression by determining access to common resources</li></ul>	<ul style="list-style-type: none"><li>• Culturally coded<ul style="list-style-type: none"><li>→ Can take <i>any</i> form, from strict dictatorship to complete egalitarianism</li></ul></li><li>• Controlled mainly by the Neocortex<ul style="list-style-type: none"><li>→ Mostly conscious</li></ul></li><li>• Its main purpose is to harmonize the behaviour of the group members via political power</li></ul>



# Hierarchy in humans

