# Bioinspired systems

## Hierarchy formation I:

## Collective motion and collective decision making

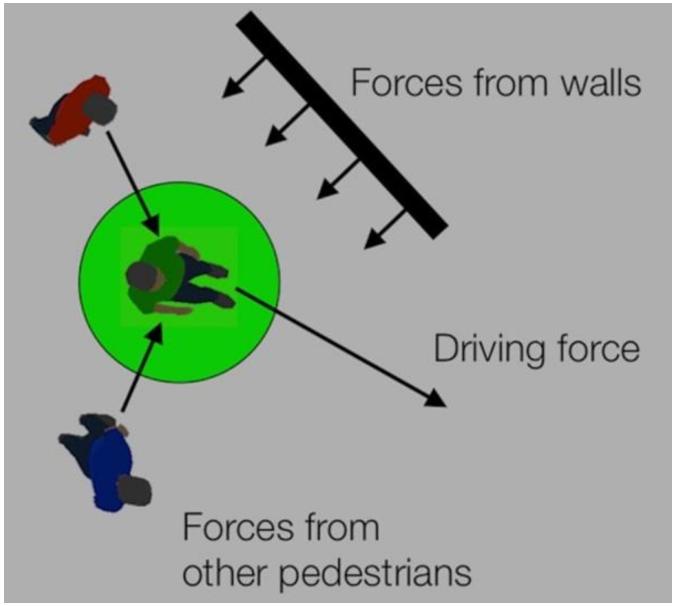
10/16/2024

# Force based models/social force models (SFM)

- A microscopic level pedeatrian CM model
- Helbing and Molnár (1995)
- People walk in crowded environments by using **automatic (subconscious) strategies** for avoiding collisions and keeping comfortable distances

- These automatic strategies can be encoded as simple behavioral rules

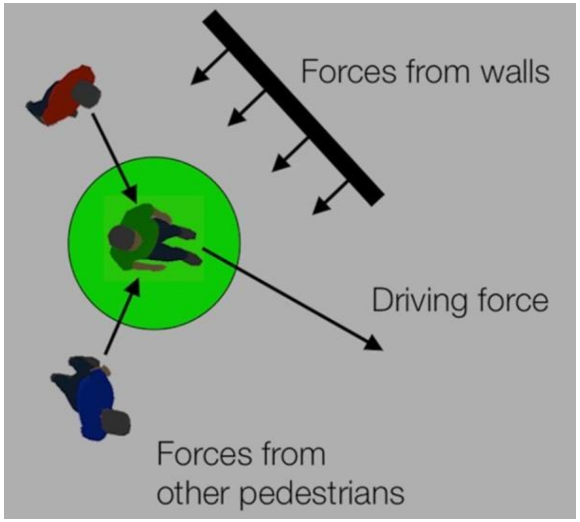
- **Social forces**:
  - **Driving force**: directs pedestrians towards their intended dectination at a desired speed
  - **Repulsive force**: prevents pedestirans from coming too close to others or to obstacles (maintaining personal space)




**Main idea**:

the influences of elements of the environment on the behavior of the pedestrians appear as social forces.

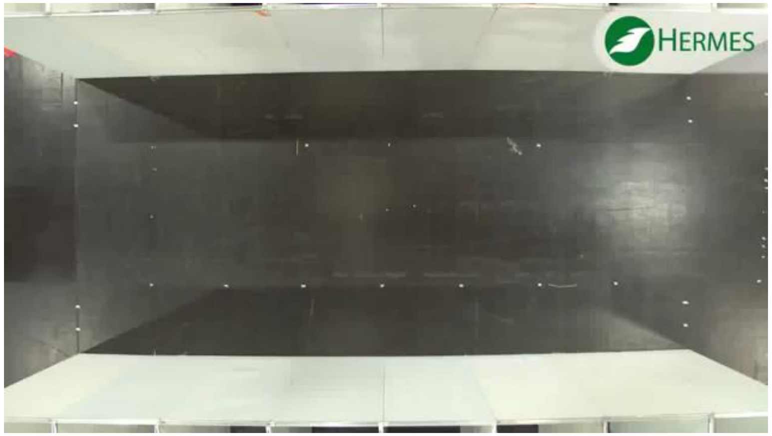
# Force based models/social force models (SFM)

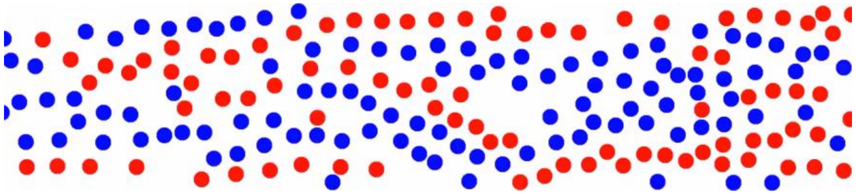



- Social forces are a description of the motivation of the pedestrian to change its velocity, induced by some elements in the environment.
- the effects of several social forces, just like regular forces, are assumed to **add as vectors**
- Operates in **continuous space**, allowing detailed representation of the geometry of the environment
- proven to reproduce several well known features of pedestrian traffic:
  - dynamic lane formation in opposing flows
  - oscillations at bottlenecks
  - evacuation scenarios

**Main idea**: the influences of elements of the environment on the behavior of the pedestrians appear as social forces.

# Dynamic lane formation in opposing flows



**Experiment:**

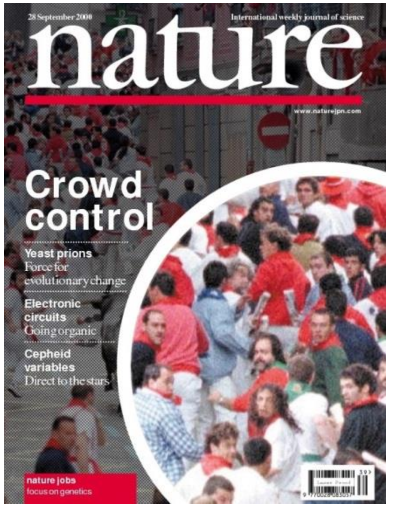
Walkers self-organize into lanes to avoid interactions with oncoming pedestrians. This helps them to move faster than is otherwise possible. This happens effortlessly and requires no communication
https://www.youtube.com/watch?v=J4J__lOOV2E

**Model:**

F. Zanlungo, T. Ikeda and T. Kanda,
Social force model with explicit collision prediction,
Europhysics Letters, Volume 93, 68005

https://www.youtube.com/watch?v=u2kEM2Ed6Xk

# An application for SFM: Panic in human crowd




According to the **socio-psychological** literature the characteristic features of **escape panics**:

(1) People try to move considerably **faster** than normal
(2) Individuals start **pushing**, and interactions become physical.
(3) Moving and passing of a **bottleneck** becomes uncoordinated.
(4) At **exits** arching and clogging are observed.
**(5) Jams** build up
(6) The **physical interactions add up** and cause dangerous pressures up to 4,450 $^{N}/_{m^2}$ which can bend steel barriers or push down brick walls

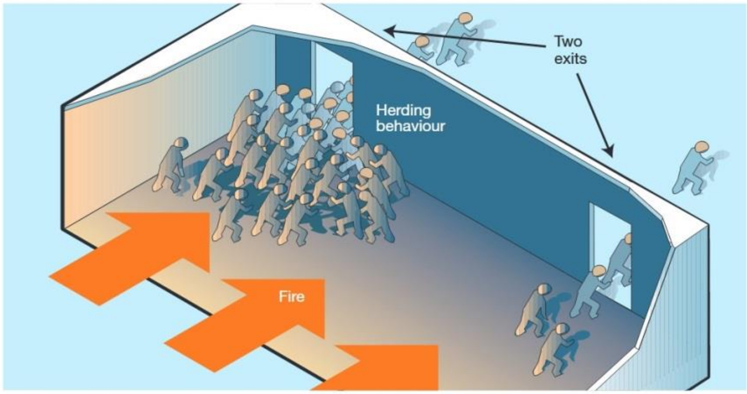
Dirk Helbing, Illés Farkas, and Tamás Vicsek: *Simulating dynamical features of escape panic*. Nature 407, 487-490 (2000)

# Model: Panic in human crowd

- Many-particle SPP system
- Main assumption: the individual behavior is influenced by a mixture of socio-psychological and physical forces

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0(t)\mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}$$

$N$: number of pedestrians (size of the crowd)
$m_i$: mass of the $i$-th pedestrian
${v_i}^0$: desired speed of individual $i$
${\boldsymbol{e}_i}^0$: preferred direction of individual $i$
$\boldsymbol{v}_i(t)$: actual velocity
$\tau_i$: characteristic („reaction”) time of individual $i$
$f_{ij}$ and $f_{iW}$: „interaction forces”: individual $i$ tries to keep a velocity-dependent distance from other pedestrians $j$ and walls $W$.




How crowd behaviour affects escape from a smoke-filled room. Previous simulations of pedestrian behaviour in crowds have used a model based on fluid flow through pipes, but these ignored the actions of individuals. According to the individual-centred model of Helbing *et al.*[1], the evacuation of pedestrians from a smoke-filled room with two exits can lead to herding behaviour and clogging at one of the exits. By contrast, a traditional fluid-flow model would predict the efficient use of both exits. A more individual-centred approach is required to reproduce the behaviour of real crowds.

# Panic model – cont.

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0(t)\mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}$$

The psychological tendency of pedestrians $i$ and $j$ to avoid each other: repulsive interaction force:

$$A_i e^{\frac{r_{ij}-d_{ij}}{B_i}} \boldsymbol{n}_{ij}$$

If $d_{ij} < r_{ij}$ then the pedestrians touch each other. In this case two additional forces (after granular interactions):

1. "Body force":

   $$k(r_{ij} - d_{ij})\boldsymbol{n}_{ij}$$

   counteracting body compression
2. "Sliding friction force"

   $$\kappa(r_{ij} - d_{ij})\Delta v_{ij}{}^t \boldsymbol{t}_{ij}$$

   impeding relative tangential motion

   $\boldsymbol{t}_{ij}$ is the tangential direction, and

   $\Delta v_{ij}{}^t = (\boldsymbol{v}_j - \boldsymbol{v}_i) \cdot \boldsymbol{t}_{ij}$ is the tangential velocity difference

$N$: number of pedestrians (size of the crowd)
$m_i$: mass of the $i$-th pedestrian
$v_i{}^0$: desired speed of individual $i$
$\boldsymbol{e}_i{}^0$: preferred direction of individual $i$
$\boldsymbol{v}_i(t)$: actual velocity
$\tau_i$: characteristic („reaction") time of individual $i$
$f_{ij}$ and $f_{iW}$: „interaction forces": individual $i$ tries to keep a velocity-dependent distance from other pedestrians $j$ and walls $W$.

$\boldsymbol{r}_i(t)$ position of individual $i$
$A_i$ constant
$B_i$ constant
$d_{ij} = \|\boldsymbol{r}_i - \boldsymbol{r}_j\|$ distance between the pedestrians' center of mass
$\boldsymbol{n}_{ij}$ : normalized vector pointing from pedestrian $j$ to $i$
$r_i$ : the radius of pedestrian $i$
$r_{ij} = r_i + r_j$ the sum of the radii of pedestrians $i$ and $j$

$\kappa$ : constant (large)
$k$ : constant (large)

$g(x)$ : zero, if the pedestrians do not touch each other ($d_{ij} > r_{ij}$),
Otherwise equal to the argument $x$.

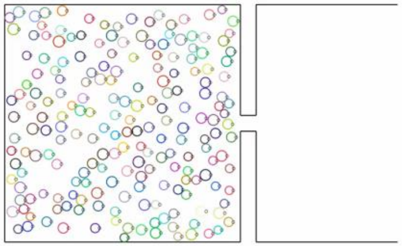
$$f_{ij} = \left\{A_i e^{\frac{r_{ij}-d_{ij}}{B_i}} + k \cdot g(r_{ij} - d_{ij})\right\} \boldsymbol{n}_{ij} + \kappa g(r_{ij} - d_{ij})\Delta v_{ij}{}^t \boldsymbol{t}_{ij}$$

# Simulation results with reasonable parameters

1. Transition to incoordination due to clogging.

The outflow from a room is well coordinated and regular desired velocities are normal.

But for desired velocities above $\mathbf{1.5}\ ^{m}/_{s}$ (rush) an irregular succession of **arch-like blockings** of the exit and **avalanche-like bunches** of leaving pedestrians when the arches break appear.

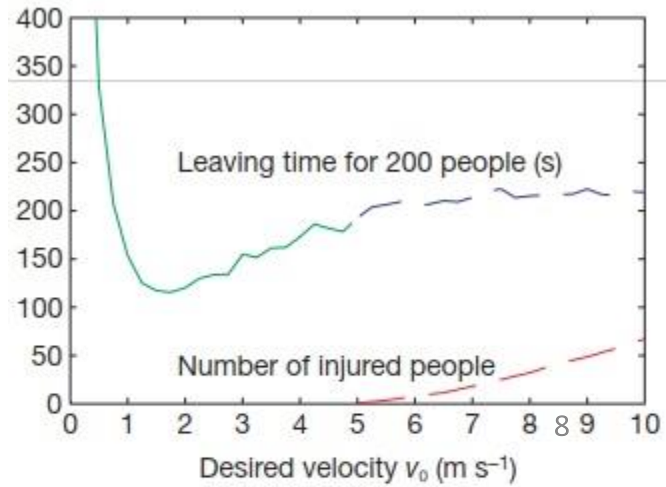


Simulation of 200 pedestrians evacuating a 15x15m room passing through a 1meter-wide door at a desired speed of 3.5m/s.

https://www.youtube.com/watch?v=FidqTZiJvRA

2. "Faster-is-slower" effect due to impatience. Since clogging is connected with delays, trying to move faster can cause a smaller average speed of leaving ($\kappa$ is large)

- fire



# Simulation results with reasonable parameters

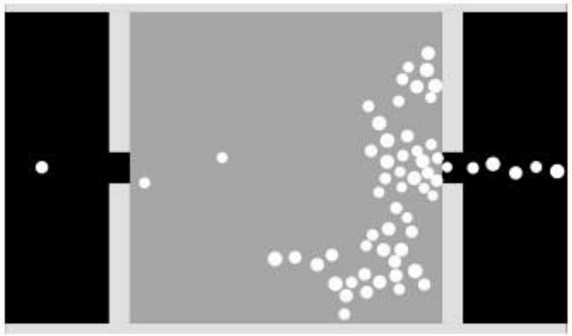
3. **Mass behavior.** Simulated situation: pedestrians are trying to leave a smoky room, but first have to find one of the invisible exits.

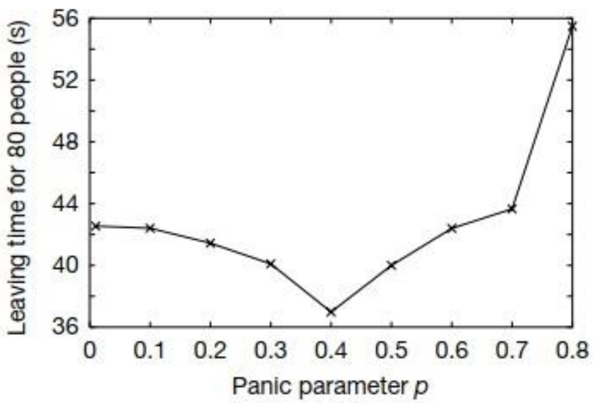
   Each pedestrian $i$ may either

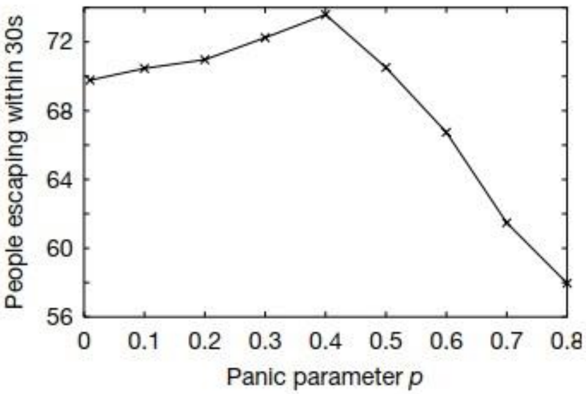
   - select an individual direction $\boldsymbol{e_i}$
   - follow the average direction $\langle \boldsymbol{e_j}^0(t) \rangle_i$ of his neighbors $j$ in a certain radius $R_i$
   - mix the two with a weight parameter $p_i$

$$\mathbf{e}_i^0(t) = \mathrm{Norm}[(1 - p_i)\mathbf{e}_i + p_i \langle \mathbf{e}_j^0(t) \rangle_i]$$

   - if $p_i$ is small → individualistic behavior
   - if $p_i$ is big → herding behavior
   - → $p_i$ is the "panic parameter" of individual $i$
   - Best chances of survival: a certain mixture of individualistic and herding behavior





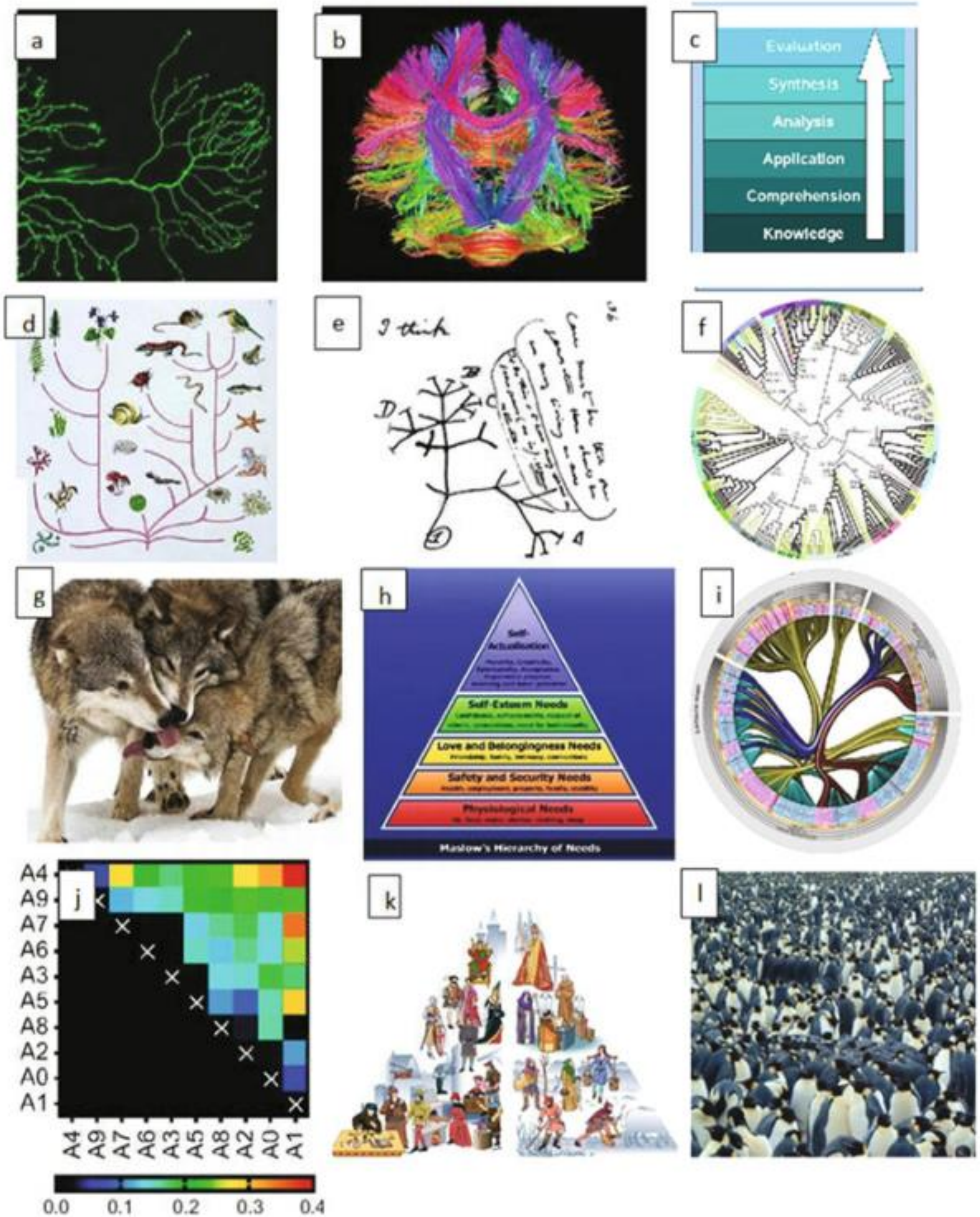
# Faster is slower in pedestrian evacuation

**Experiment (by GranularLab)**

Illustrative video experimentally demonstrating the Faster is Slower effect in pedestrian evacuation through narrow doors. The charts appearing in the vertical direction are spatio-temporal diagrams constructed by taking the lines of pixels displayed by green and stacking them vertically as time evolves. For more information: http://journals.aps.org/pre/abstract/...

https://www.youtube.com/watch?v=q-k4fCiiMlk

# Hierarchy formation
# and
# collective decision making




***a*** Axon arborisation (the end part of a major kind of neuronal cell) shows a typical hierarchical tree-like structure in space.

***b*** The wiring of a human brain. Hierarchy is not obvious, but closer inspection and additional MRI images indicate hierarchical functional operation.

***c*** And this is a possible interpretation of how we think (thoughts being one of the end products of a functioning brain).

***d*** The visualization (of the now commonplace) idea of the evolutionary tree.

***e*** The famous first drawing of the branching of the phylogenetic tree with the “I think” note by Darwin.

***f*** This complex tree with its hundreds of branches shows the birth of new variants (associated with new plant species) of a single protein!

***g*** The well-known hierarchy of wolves, indicated by who is licking who (subordinates do this to those above them). The same behavior can be observed between a dog and her owner.

***h*** Perhaps the only hierarchy named after a person. This pyramid is called “Maslov’s hierarchy of needs”.

***i*** Visualization of the connections (call relations) between the various parts of a C+ software system (containing many thousands of entities and relations; the more closely related parts are color-coded and bundled).

***j*** The strength of the directional correlations between pairs of pigeons in a flock (individuals being denoted by A0,…,A9). The asymmetric structure of the dominant part of the matrix (the entire matrix minus its symmetric components) indicates strictly hierarchical leader-follower relations.

***k*** The picturesque representation of the two pyramids of medieval relations among the member s of a society: the left side corresponding to social organization, the right side corresponding to the religious organization.

***l*** And finally: we show a huge community of relatively simple animals. Where is the hierarchy here? Nowhere: groups of many thousands of animals (large flocks of birds, schools of fish) typically do not display the signs of hierarchy (and, indeed, are assumed not to be hierarchically organized).

# Definition

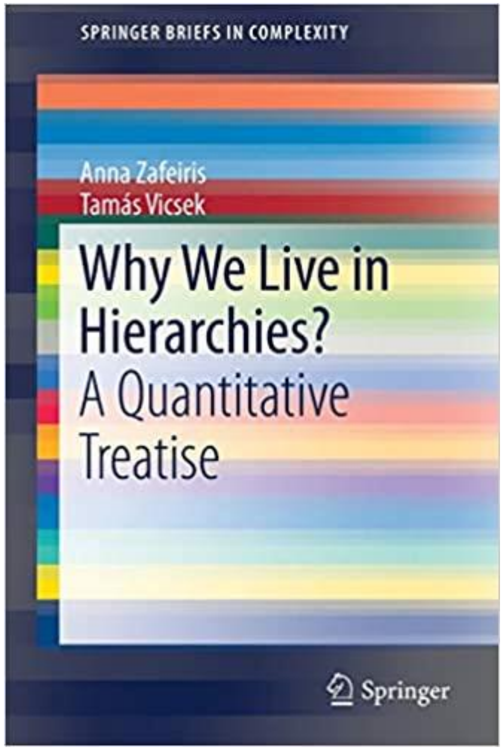
- No compact, precise, widely accepted definition
  (diverse usage)
- Available definitions usually bypass the problem of clarification by using synonymous words

- Cambridge dictionary:
  - Hierarchy is “a system in which people or things are arranged according to their importance.”
  - hierarchy corresponds to “the people in the upper levels of an organization who control it.”

- Wikipedia: “A **hierarchy** (from the Greek *hierarkhia*, "rule of a high priest", from *hierarkhes*, "president of sacred rites") is an arrangement of items (objects, names, values, categories, etc.) in which the items are represented as being "above", "below", or "at the same level as" one another.”

# Definition: hierarchy

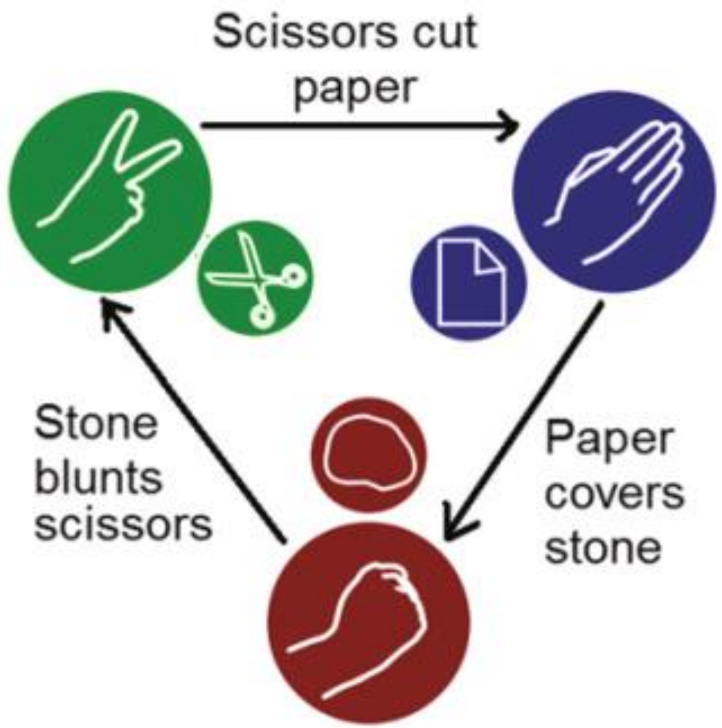
We talk about *hierarchy* in situations in which the *entities of a system can be classified into levels in a way that elements of a higher level* ***determine or constrain the behavior and/or characteristics*** *of entities in a lower level*. That is, at the heart of hierarchy, we find ***control*** *of behavior*.




**Definition**: A system is *hierarchical* if it has elements (or subsystems) that are in dominant-subordinate relation to each other. A unit is *dominant* over another unit to the extent of its ability to influence the behavior of the other. In this relationship, the latter unit is called *subordinate*.

# Comments on the definition of hierarchy - I

- It does not tell us how hierarchical the entire *system* is.
- It tells whether the *elements* (or subsystems) are in hierarchical relation or not? (manifesting itself in a dominant-subordinate relationship)
- It also tells the *origin* (reason) and *extent* of the dominant-subordinate relationship
- Rock–paper–scissors game:
  - The rock blunts the scissors (and hence "disarms" it, beats it)
  - The scissors cut the paper, and
  - The paper covers the rock.
- From a graph-theoretical point of view: where to put the arrows and what they mean there.
- It does not tell us how hierarchical the entire system is.
- "Measuring the level of hierarchy" in directed graphs has an entire literature




**Definition**: A system is *hierarchical* if it has elements (or subsystems) that are in dominant-subordinate relation to each other. A unit is *dominant* over another unit to the extent of its ability to influence the behavior of the other. In this relationship, the latter unit is called *subordinate*.

# Comments on the definition of hierarchy - II

- This definition implies that the units *behave* somehow, or have some observable characteristics. → entities without observable behavior or characteristics cannot form hierarchical structure.

- Hierarchy might vary over time.
  - As certain characteristics of the group members change (for example, the physical strength of the individuals in a pack of wolves), so do their ranks.

- During different group activities, the influence of the members might vary.

  → hierarchy is context/task-sensitive, even within the same group!
  - E.g.: pigeon flocks: Feed / collective flights.
  - even more starkly expressed in human groups

- The influence can either be
  - ***forced*** by the higher-ranked individual (e.g., when a higher-ranked animal does not let a lower-ranked one near the food source), or it can be
  - ***voluntary*** (for example, leader-follower relationships during flight).

- A higher-ranked unit, by influencing the behavior of other units more extensively, has a larger effect on the collective (emergent) group behavior as well.

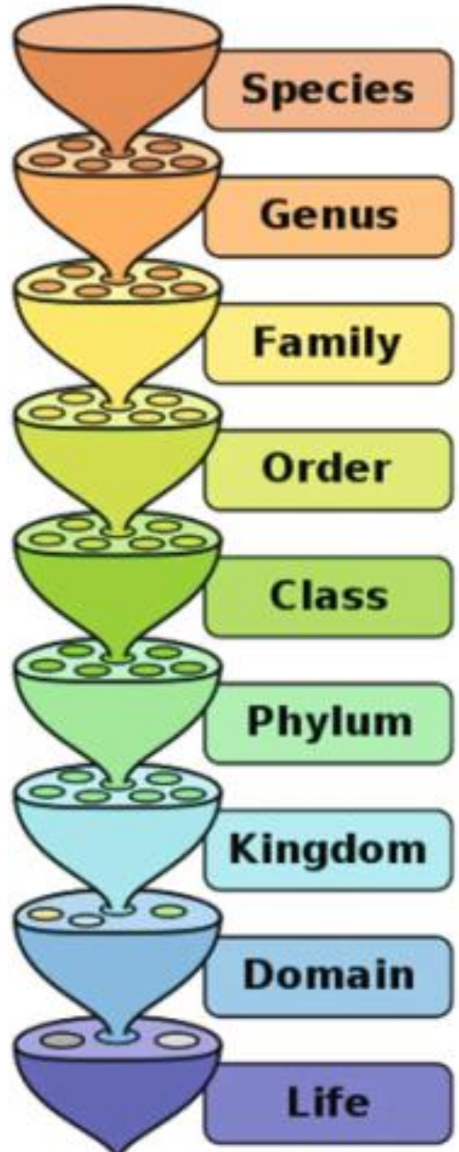
# Types of hierarchies

| Name | Description | example |
|---|---|---|
| Order hierarchy | Basically an ordered set, in which a value is assigned to each element characterizing one of its arbitrarily chosen features, which defines its rank.<br><br>The network behind the system is neglected or it does not exist. | • ranking of artists, e.g., painters or sculptors, based on the average price of their artworks<br>• firms ordered by their<br>    • number of employees<br>    • annual income, etc. |




# Types of hierarchies

| Name | Description |
|---|---|
| **Nested**<br>**Embedded**<br>**Containment**<br>**Inclusive**<br>Hierarchy | A structure in which entities are embedded into each other.<br>Higher level entities consist of and contain lower level entities.<br><br>Close relation to community detection in graphs |
| A ***subsumptive* containment** hierarchy (a.k.a. ***taxonomic hierarchy***) | A structure in which items are classified from specific to general |




C. Lupus (d dog)

Canis

Canidae

Carnivora

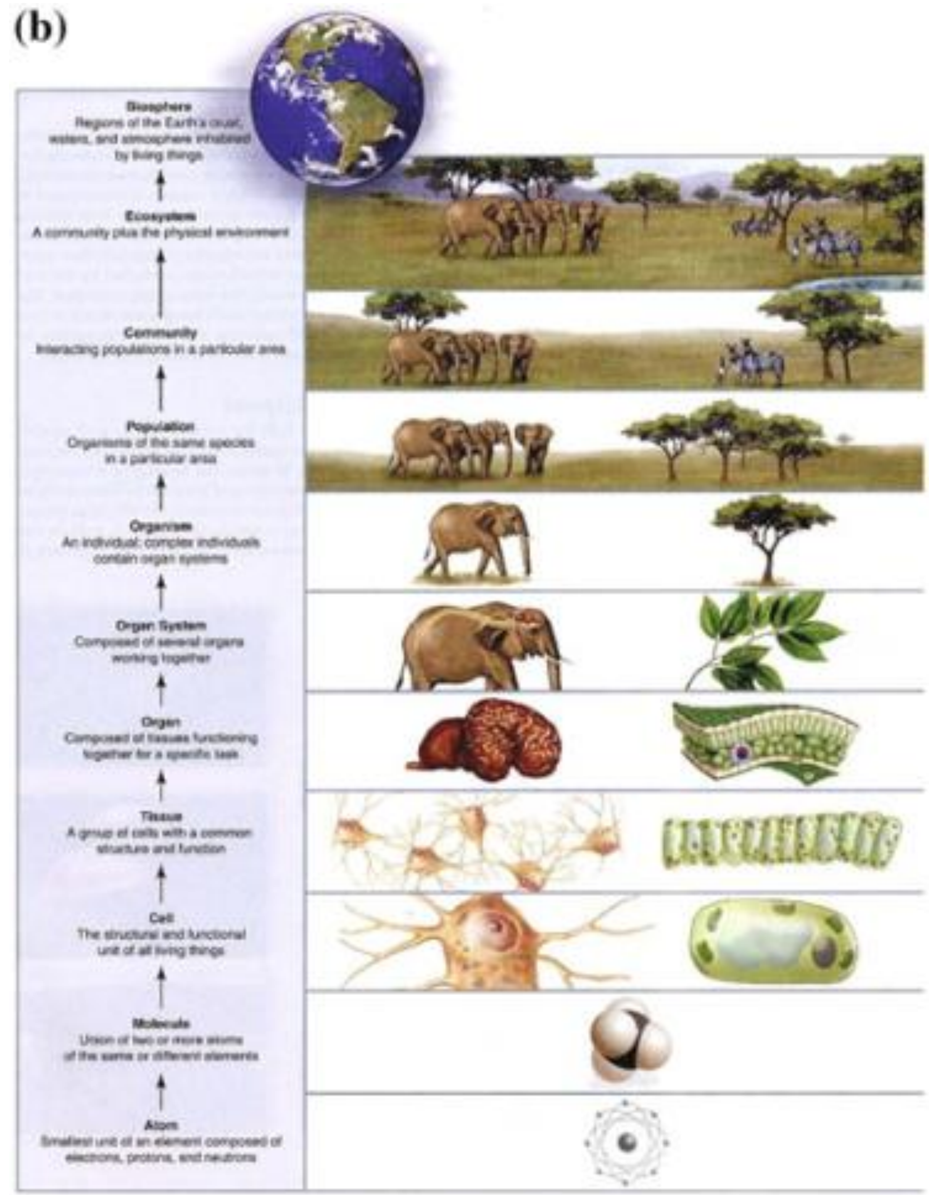
Mammalia

Chordata

Animalia

# Types of hierarchies

| Name | Description |
|---|---|
| **Nested**<br>**Embedded**<br>**Containment**<br>**Inclusive**<br>Hierarchy | A structure in which entities are embedded into each other.<br>Higher level entities consist of and contain lower level entities.<br><br>Close relation to community detection in graphs |
| A ***Compositional*** **containment** hierarchy<br>***(****a.k.a.* ***level hierarchy)*** | Describes how a system is composed of subsystems, which are also composed of subsystems, etc.<br>• “Hierarchy of life” |



# Types of hierarchies

| Name | Description | example |
| --- | --- | --- |
| **Flow** (or **control**) hierarchy | "Intuitively," this is an acyclic, directed graph. Nodes are layered into levels: nodes on higher levels influence nodes on lower levels, and the influence is represented by edges.<br>Layers refer to power, that is, an entity on a higher level gives orders or passes on information to entities on lower levels.<br>("flow of order")<br>How certain entities control other entities. | • Armies, churches, schools, political parties, institutions, etc.<br><br>• Downwards: orders flow along the edges;<br>• Upwards: requests or information. |

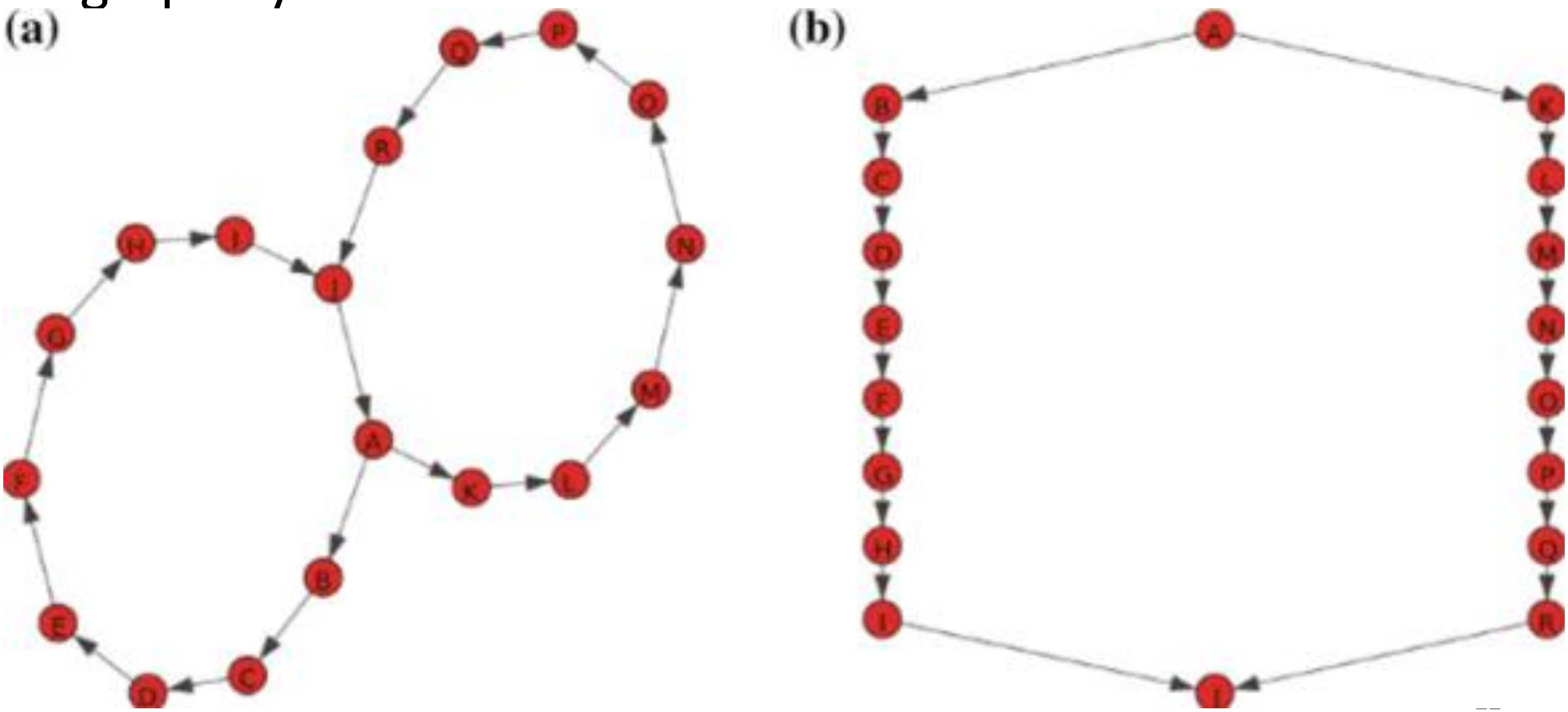
- These types are not independent of each other
- many systems can be described by more than one type (e.g. army: flow & compositional containment)
- Both order and nested hierarchies can be converted into a flow hierarchy.

# Describing hierarchical structures

- Most commonly used mathematical tool: ***graphs***
- Primarily they are connected to systems embodying *flow hierarchy*
  - observations, experiments, computer simulations are likely to return flow hierarchy;
  - all other hierarchy types can be transformed into flow hierarchy in a rather straightforward way
- We can measure the hierarchical level of the *graph* (not the system itself)
- No "most appropriate" measure (many structure is "matter of intuition / taste")
- Most of the proposed measures take values on the [0, 1] interval

# Some common approaches
# For directed and undirected graphs

- Fraction of edges participating in cycles
- Minimum fraction of edges to be removed to make the graph cycle-free



# Random Walk Measure

- Motivation:
  - it is not correct to treat all directed acyclic graphs as already being maximally hierarchical, independent of their inner structure.
  - common intuition: a hierarchical structure often corresponds to a multi-level pyramid in which the levels become more and more wide as one descends from the higher levels towards the lower ones
- Assumption: there is information/instruction flow from the high-ranking nodes towards the bottom ones
- Method:
  - find the sources by dropping down random walkers onto the nodes who then move *backwards* along the links
  - Once a steady state is reached, the *density* of such random walkers is interpreted as being proportional to the *rank* of the node:
    - high random walker density: the vertex is a *source* of information (high rank)
    - low density: the vertex is just a “receiver” of orders (low rank)
  - The hierarchical nature of the network: estimated based on the *distribution* of random walker densities
    - Homogeneous: the source of information/order cannot be pinpointed: not hierarchical
    - Inhomogeneous: clear information sources: the network is hierarchical.

Czégel D, Palla G (2015) *Random walk hierarchy measure: what is more hierarchical, a chain, a tree or a star?* Sci Rep 5:17994

# Global Reaching Centrality (“GRC”)

- Central idea: to give a rank to each node by measuring its “impact” on other nodes
  - “Impact”: the ratio of vertices that can be reached from the focal node *i* – this is the *“local reaching centrality”*
  - In a directed, un-weighted graph $C_R(i)$ is the number of vertices that can be reached from node *i*, divided by $N-1$
  - The level of hierarchy is inferred from the *distribution* of the $C_R(i)$ values
    - Heterogeneous distribution: hierarchical network
    - Homogeneous distribution: non-hierarchical graph
- From distribution to number:
  - Let $C_R^{max}$ denote the highest $C_R(i)$ value in a graph $G=(V,E)$
  - Then *GRC*, the Global Reaching Centrality is:

$$\mathrm{GRC} = \frac{\sum_{i \in V} \left[ C_R^{\max} - C_R(i) \right]}{N-1}$$

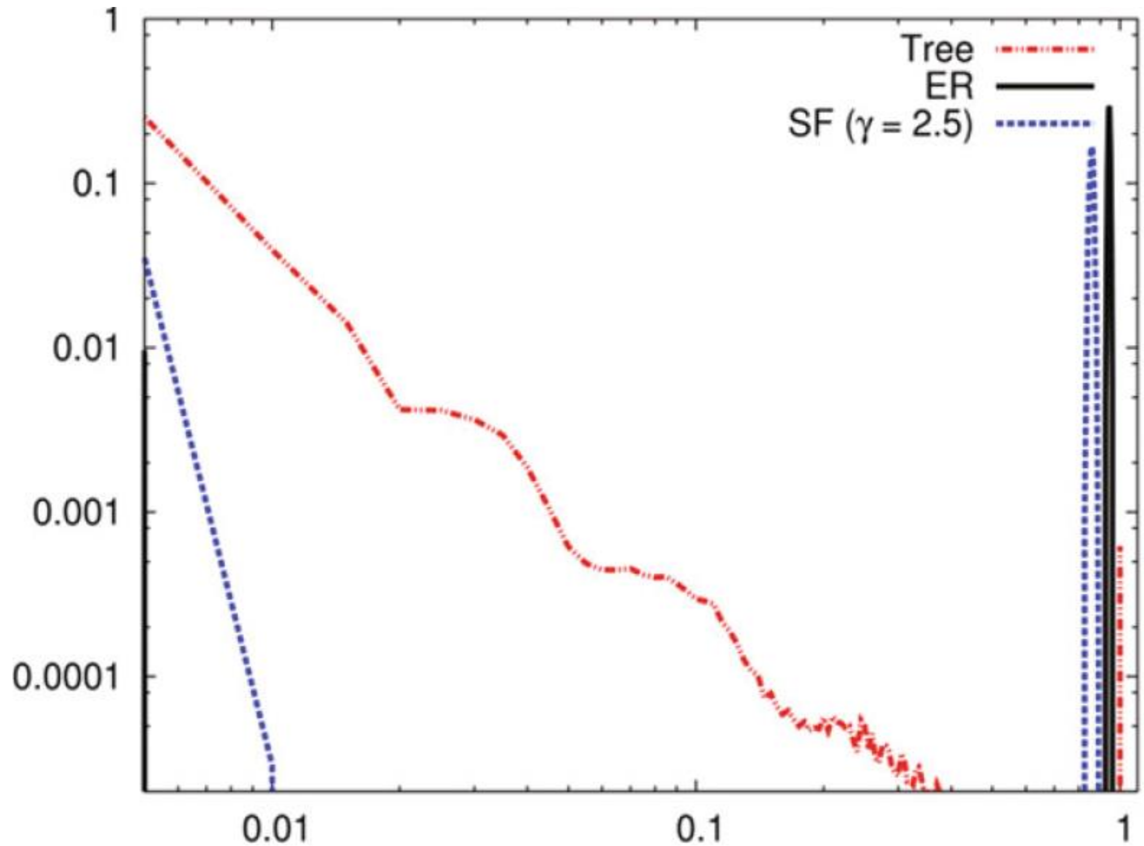
# Global Reaching centrality (“GRC”)

Example: GRC distribution for three different network types:

- Erdős-Rényi (random) (not hier)
- Scale-free (moderately hier)
- Tree (highly hier)

$$\mathrm{GRC} = \frac{\sum_{i \in V}\left[C_R^{\max} - C_R(i)\right]}{N-1}$$

| Network type | GRC |
| --- | --- |
| Erdős-Rényi | 0.058 ± 0.005 |
| Scale-free | 0.127 ± 0.008 |
| Tree | 0.997 ± 0.001 |




Distributions of the local reaching centralities for three kinds of directed network: Tree, Erdős-Rényi (ER) and scale-free (SF). All the curves are averages of *1000* graphs with *N = 2000*, of the appropriate graph type.

# Observations and measurements

# Dominance hierarchy

- Solitary vs. social lifestyles
- If the ratio of advantages/disadvantages is higher, then the given animals will knit into groups
- A mechanism is needed to reduce the level of aggression triggered by the competition
- Regulate access to resources.
- The mechanism is simple: higher ranked individuals have primacy compared to their lower level mates.
- As one advances in the evolutionary tree, the structure of the dominance hierarchy gets more and more pronounced and complex, accompanied by more and more sophisticated strategies by which individuals try to get higher and higher ranks.
- Chimpanzees (few decades ago believed to be solely human):
  - coalition formation
  - manipulation
  - exchange of social favors
  - adaptation of rational strategies
- Obvious advantage: less fight