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Computer simulations of the collective displacement of self-propelled agents

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ABSTRACT

We report extensive computer simulations of the Vicsek model [V.M. Vicsek, et al., Phys. Rev. Lett. 75 (1995) 1226], aimed to describe the onset of ordering within the low-velocity regime of the collective displacement of self-driven agents. The VM assumes that each agent adopts the average direction of movement of its neighbors, perturbed by an external noise. The existence of a phase transition between a state of ordered collective displacement (low-noise limit) and a disordered regime (high-noise limit) is most likely the most distinctive feature of the VM. In this paper, after briefly discussing the critical nature of the transition we focus our attention on the behavior of the VM in the low-velocity ($v_0 \to 0$) regime for the displacement of the agents. In fact, while the XY model, which could somewhat be considered as the equilibrium counterpart of the VM, does not exhibit order in $d = 2$ dimensions, an intriguing feature of the VM is precisely the onset of order. Since in the XY model the particles remain fixed in the lattice, we show that the understanding of the $v_0 \to 0$ limit is relevant in order to explain the different behavior of both models.

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1. Introduction

Nowadays, within the physical community, there is great interest in the study and characterization of biologically and ecologically motivated models, such as prey-predator systems [1,2], forest-fire models [3,4], simple models for the co-evolution of species [5,6], models for the collective displacement of self-driven individuals [7–11], etc.

Now, focussing our attention on the description of the collective motion of agents, the Vicsek model [7] represents an archetypical case that has extensively been studied [12–17]. The VM considers N individuals in $d = 2$ dimensions. Individuals at (off-lattice) positions $\vec{x_i}$ have velocities $\vec{v_i}$ and move in the direction θ_i, here $i = 1, 2, \ldots, N$. In order to account for the self-propelled nature of the motion, the magnitude of the velocity is fixed at v_0 for all individuals. Individuals interact locally by trying to align their directions of motion with that of their neighbors, in the presence of some perturbation (noise). This rule is implemented by assuming that at each time step, a given individual assumes the average direction of motion of the individuals located within its local neighborhood (a circle of radius R_0), namely

$$\theta_i(t + \Delta t) = \langle \theta(t) \rangle_{R_0} + \xi_i(t), \tag{1}$$

where the noise (ξ) has been introduced as a random variable with uniform distribution in the interval $[-\eta\pi, \eta\pi]$, and the local average direction of motion $\langle \theta(t) \rangle_0$ is defined as the average direction of the velocities of individuals (including the ith one) within the radius of interaction R_0. Also, the locations of the individuals are updated in each time step according to

$$\vec{x_i}(t + \Delta t) = \vec{x_i}(t) + \vec{v_i}(t)\Delta t. \tag{2}$$

The interest in the VM further increases because, very recently, Chaté et al. [16] have cast doubt on the critical nature of the transition that was early reported [7], since they claim that the transition should actually be of first order. This topic leads to an ongoing controversy on view of both the results of Aldana et al. [18], and our recent simulations of the dynamical critical behavior [17] that also support the occurrence of a second-order transition in the VM and related models.

The analogy of the VM with simple ferromagnetic models (Ising, XY, etc.) is a topic that has also originated interesting discussions. In fact, by considering that the Hamiltonian tending to align the spins in the same direction is replaced by the tendency of the individuals to align in the same direction (Eq. (1)), and the amplitude η of the random perturbation ξ plays the role of the temperature, one should expect the observation of some qualitatively similar features when comparing these systems. For example, within the low-velocity regime of the VM ($v\Delta t \ll 1$), it has been shown that individuals tend to move in an ordered fashion when the noise is decreased below a (particle density (ρ)) critical

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value ($\eta_c(\rho)$). In contrast, for $\eta > \eta_c(\rho)$ one observes a regime of disordered motion without net mass transport. Therefore, even in $d = 2$ dimensions, the VM exhibits an order–disorder phase transition [7]. However, it is worth mentioning that the VM is a far-from equilibrium system that obeys a quite different dynamics from that of standard ferromagnetic models [19,20].

Nevertheless, the major difference between the VM and a typical magnetic model, e.g., the XY model, is that the former involves the displacement of the particles, while in the latter the spins are placed at fixed positions in a lattice. Under these circumstances it would be very useful to understand the dependence of the onset of ordering in the VM upon changing the velocity of the particles. In fact, one would expect that somewhat in the limit $v_0 \to 0$ the ordered phase of the VM would vanish.

Within this context, the goal of this paper is twofold. First, we will briefly discuss our evidence on the critical nature of the order–disorder transition of the VM. Subsequently, extensive simulations aimed to clarify the behavior of the VM in the low-velocity limit will be presented and discussed. This latter topic is a subject of current interest because particle motion seems to be the most relevant and distinctive feature capable of inducing ordering in the VM, as compared with the XY model that does not exhibit an ordered phase in $d = 2$ dimensions.

The manuscript is organized as follows: in Section 2 we provide a brief description of technical details of the simulation method and define some relevant observables. The obtained results are presented and discussed in Section 3. Finally, we state our conclusions in Section 4.

2. Description of the simulation method and definition of relevant observables

The numerical simulations of the VM are performed in $d = 2$ dimensions by using samples of side L and assuming periodic boundary conditions. As mentioned in the Introduction, off-lattice displacement of the individuals is considered. Also, in order to avoid artifacts arising from rather large jumps of the particles, we restrict ourselves to studying the low-velocity regime by taking $v_0 \leqslant 0.1$, or in other words, our simulations are performed for $v_0 \Delta t \leqslant R_0$ (see Eq. (1)). It should also be mentioned that test runs performed by taking $v_0 \gg 0.5$ already produce undesired artifacts.

In order to speed out the simulations, we implemented an algorithm already used for the simulation of fluids [21], which is based on the partition of the two-dimensional sample in a regular network of M cells, each of them of side $l = L/M^{1/2}$. In our simulations, the number of cells is defined by the interaction distance among particles, so that one has $l = R_0$. Since the algorithm keeps track of the location of the particles inside the cells, in order to evaluate the interactions of a selected particle with its neighbors, one only needs to evaluate the neighboring cells. In fact, particles placed in the remaining cells are, of course, out of the interaction range. By using this algorithm, the evaluation of the interaction of a single particle requires us to perform the order of about $4.5N^2/M^{1/2}$ operations, as compared to $N(N-1)/2$ that are necessary in a straightforward calculation.

Simulations are performed for three different densities of individuals ($\rho = 1/8$, $1/4$, and $3/4$), and by using samples of different size ($52.26 \leqslant L \leqslant 565.69$), which involves ($2048 \leqslant N \leqslant 40\,000$) individuals. During a Monte Carlo time step (mcs) all individuals are updated once, on average. Measurements within the stationary regime and for $v_0 = 0.1$ are often performed after disregarding 5×10^5 mcs in order to avoid memory effects of the initial, randomly generated configurations. For $v_0 < 0.1$ we used $N = 4096$ individuals with $L = 128$, which corresponds to $\rho = 1/4$. Since the thermalization time required by the system in order to reach stationary conditions depends on v_0, we actually recorded time series of the relevant observables, so that along each simulation we can control the achievement of both the true asymptotic regime and the proper statistic, before interrupting the computer job. Then measurements are often performed after disregarding up to 10^7 mcs for the lowest velocities used.

The natural order parameter suitable to describe the collective behavior of the individuals is the normalized average velocity [7], given by

$$\varphi \equiv \frac{1}{N v_0} \left| \sum \vec{v}_i \right|. \tag{3}$$

In fact, for individuals moving almost randomly one has $\varphi \sim 0$, whereas, when all individuals tend to move in the same direction, one has $\varphi \to 1$.

As anticipated in the Introduction, the VM exhibits order–disorder transitions, and at criticality, the order parameter is expected to behave, as in the case of standard second-order transitions, according to

$$\varphi \sim \left[\eta_c(\rho) - \eta\right]^{\beta}, \tag{4}$$

where β is the order parameter critical exponent, and $\eta_c(\rho)$ is the (particle density (ρ) and velocity dependent) critical noise.

In addition, another useful observable in the study of equilibrium critical behavior is the susceptibility that, according to fluctuation–dissipation, can be obtained by measuring the variance of the order parameter [22]. Of course, the VM model describes a far-from-equilibrium system that no longer obeys fluctuation–dissipation. However, the fluctuations of the order parameter given by

$$\chi = \mathrm{Var}(\varphi) L^2, \tag{5}$$

with

$$\mathrm{Var}(\varphi) \equiv \langle \varphi^2 \rangle - \langle \varphi \rangle^2, \tag{6}$$

where $\mathrm{Var}(\varphi)$ is the order parameter variance and $\langle\rangle$ denotes averages over configurations, still is a useful quantity for the description of non-equilibrium systems [23–25].

3. Results and discussion

Early numerical measurements and theoretical arguments strongly suggest that the dependence of the critical noise corresponding to different densities of particles can be scaled after proper renormalization according to [12,13]

$$\eta^* = \eta \sqrt{\rho}. \tag{7}$$

Now, we can go one step further by showing that actually the probability distribution of the order parameter (pdf) can be collapsed in a single curve (see Fig. 1). Consequently, all physically meaningful moments of the order parameter, including of course the susceptibility and Binder's cumulants, exhibit universal features within the low-density regime, allowing us to focus the numerical effort on the behavior of a single density, without losing generality. Also, the pdf of the order parameter shown in Fig. 1 exhibits a single peak that becomes broader when increasing the scaled noise, as expected for the behavior of an order parameter describing a critical transition.

Fig. 2(a) shows plots of the dependence of φ on η, as obtained for three different densities and a wide range of the number of individuals. Here, we observed the rounding and shifting of the order parameter, as typically expected for systems exhibiting second-order transitions. The critical nature of the transition is further supported by the observed (not shown here for the sake of space) divergence of χ with the system size [17].

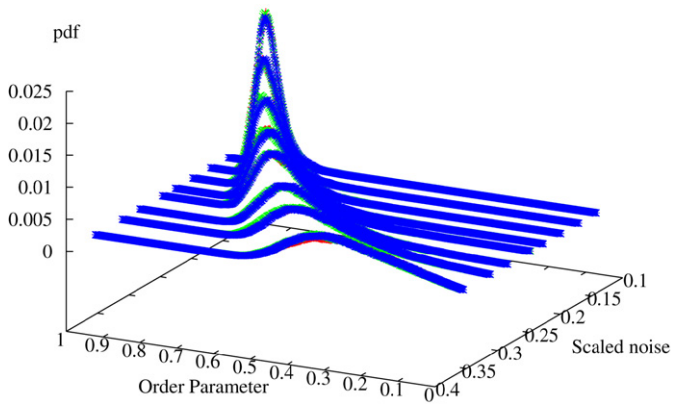

Fig. 1. (Color online) Plot of the probability distribution of the order parameter (pdf) versus both φ and the scaled noise amplitude (η^*, given by Eq. (7)). Results obtained within the stationary regime for samples with three different densities of individuals, (+) $\rho = 0.125$, (x) $\rho = 0.333$ and (∗) $\rho = 0.500$, which for $N = 8192$ individuals correspond to lattices of size $L = 256$, $L = 156.8$ and $L = 128$, respectively. More details in the text.

It is well known that in numerical simulations performed by using finite samples of linear size L, second-order phase transitions exhibit rounding and shifting effects. This shortcoming can be overcome by performing a finite-size scaling analysis. In fact, the scaling theory [26–28] established that the scaling Ansatz for the order parameter can be written as

$$\varphi(\eta, L) = L^{-\beta/\nu}\widetilde{\varphi}\big((\eta - \eta_c)L^{1/\nu}\big), \tag{8}$$

where ν is the correlation length critical exponent and $\widetilde{\varphi}$ is a suitable scaling function.

Fig. 2(b) shows the best collapse of the data corresponding to the dependence of the order parameter on the noise, as plotted in Fig. 2(a), which is obtained by taking $\beta/\nu = 0.275(5)$, and $\nu = 1.63(3)$ [17]. It is worth mentioning that, based on the results shown in Fig. 1, we obtained the nice collapse shown in Fig. 2(b) just by rescaling the noise according to Eq. (7). From the relationship $\beta/\nu = 0.275(5)$ and the obtained value of ν, one gets $\beta = 0.45(3)$. This figure is in agreement with previous simulation results that are consistent with β-values smaller than the mean-field value given by $\beta = 1/2$, namely $\beta = 0.45 \pm 0.07$ [7] and $\beta = 0.42 \pm 0.03$ [12]. Furthermore, by means of a combined study involving both stationary and dynamic measurements, we have recently determined the complete set of exponents of the VM, e.g., obtaining $\gamma = 2.3(4)$ [17] for the susceptibility exponent such that $\chi \propto (\eta - \eta_c)^{-\gamma}$. All these exponents are consistent with the hyperscaling relationship $d\nu + 2\beta = \gamma$, which not only is well established in the field of equilibrium critical phenomena but also holds for the VM under far-from equilibrium conditions. It is worth mentioning that recently, the critical nature of the transition of the VM has been challenged by Chaté and Grégoire [16], who claim that the transition should actually be of first order. Subsequently, Vicsek et al. [14] have preliminary clarified the issue by demonstrating that the presence of an inherent numerical artifact strongly influences the results of Chaté et al. [16], preventing a meaningful physical interpretation of the results.

Now, after establishing the critical nature of the order–disorder transition of the VM, we focus our attention on its dependence on the velocity of the agents. Fig. 3(a) shows plots of the order parameter as a function of the noise as obtained for different velocities ($0.005 \leqslant v_0 \leqslant 0.1$). Here one observes that for both low ($\eta \leqslant 0.10$) and high noises ($\eta \geqslant 0.3$) the order parameter is rather independent of v_0. However, for an intermediate regime, i.e., close to $\eta \simeq 0.2$, it is possible to observe a subtle systematic deviation of the points, such that φ tends to increase when v_0 is decreased. This subtle behavior of the order parameter becomes more evident

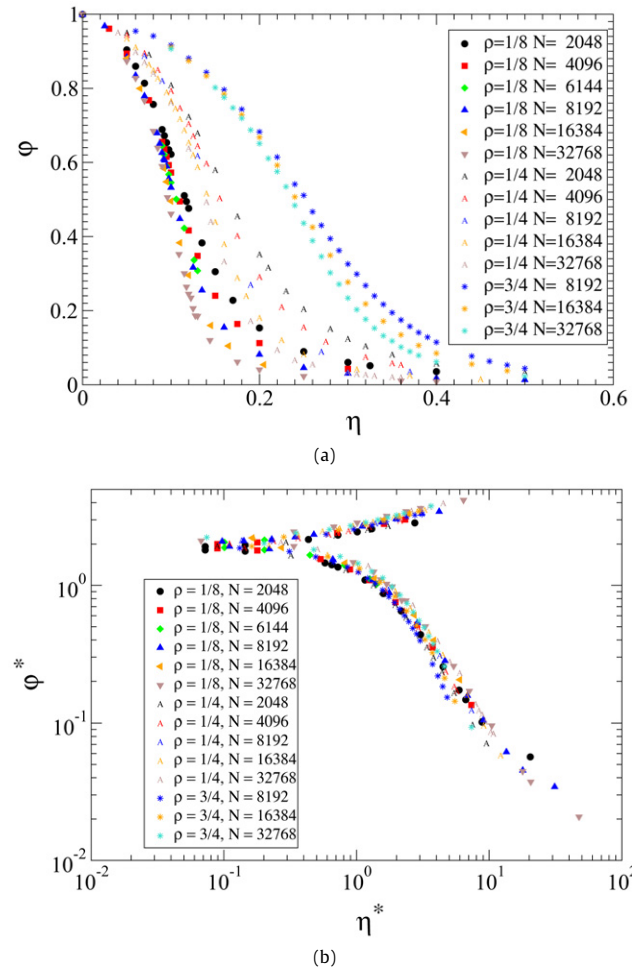

Fig. 2. (Color online) (a) Plot of the order parameter (φ) versus the noise amplitude (η). Results obtained within the stationary regime for samples of different size and by varying the number of individuals, as listed in the figure. (b) Finite-size scaling analysis of data shown in (a), performed according to Eq. (8), showing log–log plots of the rescaled order parameter $\varphi^* = \varphi(L)L^{\beta/\nu}$ versus the density-rescaled noise $\eta^* = \frac{|\eta(\rho)-\eta_c(\rho)|}{\sqrt{\rho}}L^{1/\nu}$. More details in the text.

upon measuring its fluctuations (see Eq. (5)), as shown in Fig. 3(b). In fact, while for $\eta \geqslant 0.3$ the behavior of χ is almost independent of v_0, for $\eta < 0.25$ one observes a systematic shift of the peaks towards smaller noises when the velocity is decreased.

Now, let us recall that one often identifies the location of the maximum of the fluctuations of the order parameter with size-dependent (pseudo) critical points ($\eta_c(L)$), which are subsequently used to extrapolate the data to the thermodynamic limit [17]. By using this procedure with the results shown in Fig. 3(b) we obtained the dependence of $\eta_c(L)$ on v_0, as shown in Fig. 3(c). It is worth mentioning that, since within the low-density and low-velocity regime, the universality class of the VM is unique [17], the determination of $\eta_c(L)$ for a single sample size suffices for our purposes, so that one does not need to perform extrapolations to the thermodynamic limit in order to determine the dependence of the critical noise on the velocity, as shown in Fig. 3(c).

In Fig. 3(a) we have also included data corresponding to $v_0 = 0$, which is averaged over different initial configurations, in order to show the absence of ordering in the two-dimensional VM. Furthermore, in Fig. 3(c) we observe that the extrapolation to the $v_0 \to 0$ limit gives $\eta_c^* \simeq 0.137(5)$. So, according to this extrapolation procedure one might expect the onset of ordering in the $v_0 \to 0$ limit, however Fig. 3(a) conclusively shows that this is not the case for $v_0 = 0$.

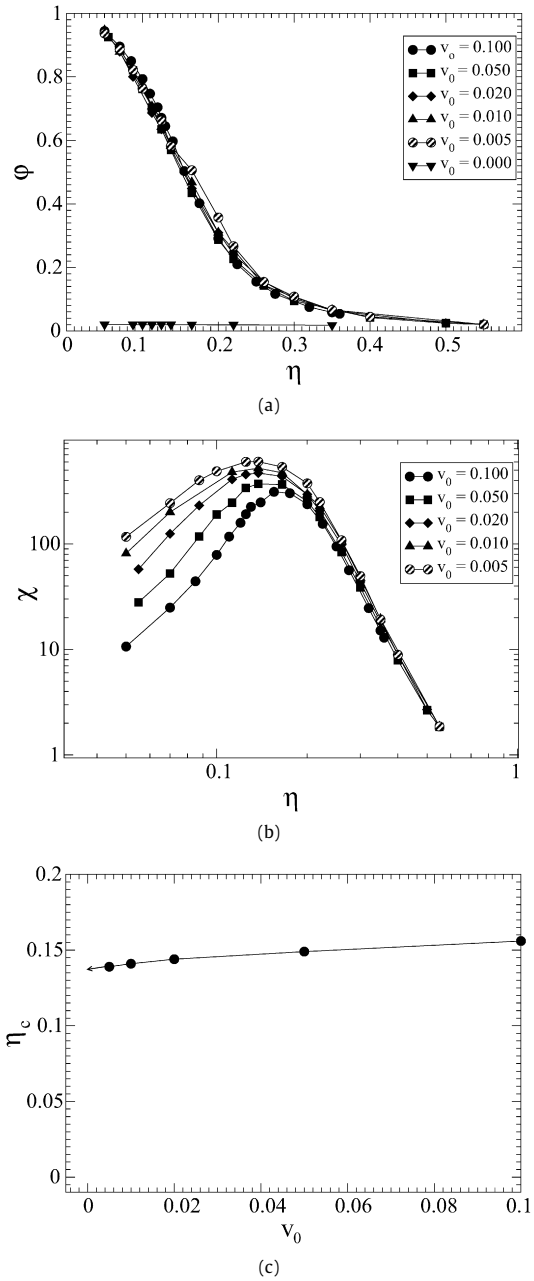

Fig. 3. (a) and (b) show plots of the order parameter (φ) and the susceptibility (χ) versus the noise amplitude (η), respectively. Results obtained, within the stationary regime, by varying the velocity of the individuals, as listed in the figures. Data corresponding to samples of side $L = 128$ ($N = 4096$). (c) Plot of the noise at the location of the peak of χ, taken from (b), versus the velocity of the individuals. The arrow shows the $v \to 0$ limit, which differs from the data point corresponding to $v = 0$. More details in the text.

4. Conclusions

We performed extensive Monte Carlo simulations of the VM for the collective displacement of self-propelled individuals, aimed to contribute to the understanding of the role of the velocity of the agents in the onset of order.

First, we established that our results are fully consistent with the critical nature of the transition, in agreement with other numerical results [7,14,17], but in contrast to the claims of Grégoire and Chaté [16] on the first-order behavior of the transition. Also, by showing the collapse of the probability distribution of the order parameter (Fig. 1) by properly rescaling the noise according to Eq. (7), we conclude that the critical behavior is the same for all densities, of course, within the low-density and low-velocity regime, a fact that allow us to focus our numerical effort on the study of the behavior of the system for a single value of the density, without losing generality.

It has been argued that the VM can be somewhat considered a non-Hamiltonian version of the well-known XY model [7], since the VM presents almost the same symmetry properties as the XY model. Of course, a major difference is that the VM involves the off-lattice displacement of the particles, while in the XY model the spins remain at fixed positions in a lattice. Also, in contrast to the XY model, the VM exhibits order in $d = 2$ dimensions.

On the other hand, it is also useful to compare our results on the VM with other out-of-equilibrium systems. For example, very recently, Wood et al. [29] have reported that a non-equilibrium (on-lattice) model for stochastic coupled oscillators, which formally can be described with the same order parameter as the VM, exhibits dimensionality dependent phase transitions. In fact, $d = 2$ is the lowest critical dimension for the observation of long-range order, and in $d = 3$ the model undergoes a continuous phase transition displaying signatures of the XY equilibrium universality class [29]. Again, a remarkable difference is that, in contrast to the VM, oscillators are placed at fixed positions.

Our results show that the behavior of the VM in the $v_0 \to 0$ limit is compatible with the onset of ordering (Fig. 3(c)), which is no longer observed exactly at $v_0 = 0$ (Fig. 3(a)). Due to this evidence, we conclude that the coupling between orientation and displacement of particles capable of changing their orientation due to interaction with other particles is an essential ingredient for the onset of ordering in the VM. Furthermore, based on preliminary results we expect that the off-lattice nature of the VM may not be a key feature responsible for the occurrence of an ordered phase. Finally, it is worth mentioning that other cellular automata statistical systems based on biological motivations, e.g., forest-fire models [3], prey-predators systems [2], the stochastic Game of Life [30], etc., exhibit analogous behavior, namely the system in the limit of a parameter going to 0 is different from the system where this parameter is exactly 0.

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References

[1] N. Boccara, O. Roblin, M. Roger, Phys. Rev. E 50 (1994) 4531–4541.
[2] A.F. Rozenfeld, E.V. Albano, Phys. Rev. E 63 (2001) 064907.
[3] E.V. Albano, Forest Fire, in: A. Scott (Ed.), Encyclopedia of Nonlinear Sciences, Routledge, New York, 2005.
[4] S. Clar, B. Drossel, F. Schwabl, J. Phys. Cond. Matt. 8 (1996) 6803; E.V. Albano, J. Phys. A (Math. and Gen.) 27 (1994) L881.
[5] M. Paczuski, S. Maslov, P. Bak, Phys. Rev. E 53 (1996) 414.
[6] K. Laneri, A.F. Rozenfeld, E.V. Albano, Phys. Rev. E 72 (2005) 065105.
[7] T. Vicsek, A. Czirók, E. Ben-Jacob, O. Shochet, Phys. Rev. Lett. 75 (1995) 1226.
[8] E.V. Albano, Phys. Rev. Lett. 77 (1996) 2129.
[9] J. Toner, Y. Tu, S. Ramaswamy, Ann. of Phys. 318 (2005) 170244.
[10] I.D. Couzin, J. Krause, N.R. Franks, S.A. Levin, Nature 433 (2005) 513516.
[11] J.M. Belmonte, G.L. Thomas, L.G. Brunnet, R.M. de Almeida, H. Chaté, Phys. Rev. Lett. 100 (2008) 248702.
[12] A. Czirók, H.E. Stanley, T. Vicsek, J. Phys. A 30 (1997) 1375.
[13] A. Czirók, A.L. Barabasi, T. Vicsek, Phys. Rev. Lett. 82 (1999) 209.
[14] M. Nagy, I. Daruka, T. Vicsek, Physica A 373 (2007) 445454.
[15] C. Huepe, M. Aldana, Phys. Rev. Lett. 92 (2004) 168701.
[16] G. Grégoire, H. Chaté, Phys. Rev. Lett. 92 (2004) 025702; H. Chaté, F. Ginelli, G. Grégoire, F. Raynaud, Phys. Rev. E 77 (2008) 046113; H. Chaté, F. Ginelli, G. Grégoire, F. Peruani, F. Raynaud, Eur. Phys. J. B 64 (2008) 451.
[17] G. Baglietto, E.V. Albano, Phys. Rev. E 78 (2008) 021125.
[18] M. Aldana, V. Dossetti, C. Huepe, V.N. Kenkre, H. Larralde, Phys. Rev. Lett. 98 (2007) 095701; J.A. Pimentel, M. Aldana, C. Huepe, H. Larralde, Phys. Rev. E 77 (2008) 061138.

[19] J.J. Binney, N.J. Dowrick, A.J. Fisher, M.E.J. Newman, The Theory of Critical Phenomena: An Introduction to the Renormalization Group, Oxford University Press, Oxford, 1992.
[20] N.D. Goldenfeld, Lectures on Phase Transitions and the Renormalization Group, Addison-Wesley, 1992.
[21] M.P. Allen, D.J. Tildesley, Computer Simulation of Liquids, Oxford University Press, Oxford, 1991.
[22] K. Binder, D.P. Landau, A Guide to Montecarlo Simulations in Statistical Physics, Cambridge University Press, Cambridge, 2002.
[23] E.V. Albano, G. Saracco, Phys. Rev. Lett. 88 (2002) 145701.
[24] S.W. Sides, P.A. Rikvold, M.A. Novotny, Phys. Rev. Lett. 81 (1998) 834.
[25] V. Manias, J.M. Candia, E.V. Albano, Eur. Phys. J. B 47 (2005) 563.
[26] M.N. Barber, in: C. Domb, J.L. Lebowitz (Eds.), Phase Transitions and Critical Phenomena, vol. 8, Academic, New York, 1983, p. 146.
[27] V. Privman (Ed.), Finite Size Scaling and Numerical Simulations of Statistical Systems, World Scientific, Singapore, 1990.
[28] J. Cardy, Scaling and Renormalization in Statistical Physics, Cambridge University Press, Cambridge, 1996.
[29] K. Wood, C. Van den Broeck, R. Kawai, K. Lindenberg, in: O. Descalzi, O. Rosso, H.A. Larrondo (Eds.), Nonequilibrium Statistical Mechanics and Nonlinear Physics, XV Conference, AIP Conf. Proc. 913 (2007) 49.
[30] R.A. Monetti, E.V. Albano, Phys. Rev. E 52 (1995) 5825.