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Information at the edge of chaos in fluid neural networks

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Abstract

Fluid neural networks, defined as neural nets of mobile elements with random activation, are studied by means of several approaches. They are proposed as a theoretical framework for a wide class of systems as insect societies, collectives of robots or the immune system. The critical properties of this model are also analysed, showing the existence of a critical boundary in parameter space where maximum information transfer occurs. In this sense, this boundary is in fact an example of the "edge of chaos" in systems like those described in our approach. Recent experiments with ant colonies seem to confirm our result.

1. Introduction

Complex dynamics -including computation- has been shown to emerge at the edge of the transition between order and chaos [1,2]. In nonlinear dynamical systems [3], cellular automata [2] and large-scale coevolution in boolean networks [4], such a transition domain is observed involving the emergence of new structural and dynamical properties. In all of these models, order appears to be a compromise between two antagonists [5]: the nonlinear process where fluctuations are strongly but coherently amplified; and the communication (transport, diffusion, etc) process, which captures, relays and stabilizes the signals.

In all the situations mentioned above, the existence of a transition zone between two clearly different types of dynamical regimes has important implications. In relation with computation, the so called *edge of chaos* has been suggested by Langton as the domain where complex computational abilities emerge [1,2], close to a second-order phase transition.

An example where self-organization and information transfer (communication) takes place is the case of social insects. Insect societies are a particular (though very extended) class of social organization. The global patterns of behavior are the result of emergent phenomena not reducible to the properties of individuals [6]. Recently such emergent behaviour has been reported in relation with chaotic dynamics in *Leptothorax* ant colonies [7,8]. Global oscilations of colony activity were reported together with the observation that individual behavior can be characterized by means of low-dimensional strange attractors [9]. The discovery of activity pulses suggests that patterns of organization occur within the ant colony as a single entity.

In earlier studies, it was observed that the ant colony as a whole can make computations not allowed to single ants [10]. To decide which of two food sources A and B is better (i.e. if the sucrose concentration C is such that $C_A > C_B$ or $C_A < C_B$, see Ref. [5]), the colony (and not the individual ants) can decide by means of symmetry breaking [11]. The asymmetry of this decision results from the interaction between the random and deterministic aspects of the communication involved [10]. Here phase transitions are used for computation in a set of essentially simple elements, which can be called *automata*. As Langton pointed out [2]: "one could use an ant colony as the model for a variant form of cellular automaton, one in which each individual cell is mobile and can move about semiautonomously. Each cell would change state by virtue of the states of the other cells in its immediate neighborhood".

The present study deals with a large class of systems characterized by the intrinsic fluidity (elements move in space or their connections are not fixed in time) and by the existence of neural-like interactions. Social insects, collectives of robots, the immune system as well as some human groups can match this description (Fig. 1). All these systems are roughly



Fig. 1. Fluid neural network. (A) A set of automata can move to nearest positions and interacts with nearest automata. Black circles are inactive elements ($S_i < \theta$) otherwise are white. (B) Several real systems can be modelized in this way, as ant colonies.

characterized by: (a) more or less large number of elements; (b) some kind of individual dynamical patterns (when isolated), eventually complex; (c) information transfer when elements interact, being the individual behaviour modified through the interaction and (d) robustness of the global pattern.

In these systems, the "local field" acting on a given element, i.e. the sum over external inputs generated from other elements, can change (eventually switch [12]) the state of such element. But this change will be always bounded in some way and, in this sense, a sigmoidal-like response can be used. If such modulation is involved and a finite set of states is available to each element, the application of the neural network (NN) formalism [13] seems justified. If no movement is present, and with a suitable choice of connections, then such a system can perform as an associative memory [13]. But if the constraint of frozen elements and fixed connections is removed, the previous possibility of associative memory based on a fixed connectivity matrix vanishes.

The aim of this paper is to explore the dynamical properties of this type of systems through a new class of formal approach, which we call *fluid neural networks*. In section 2 the model is described together with some general results. In section 3 the macroscopic behaviour and phase space is studied, showing the appearance of a critical boundary between the ordered and chaotic regimes and in section 4 the existence of computation at the edge of chaos is analysed by means of the information transfer among elements.

2. Fluid neural network

Here we will define our model, the so called fluid neural network (FNN). Our goal is to obtain the simplest model able to match the previous set of conditions. As a theoretical framework, the standard approach of neural networks is used, but a new set of rules defining local movement and individual activation are also introduced.

A set of N automata is used. The state of each automaton (say the *i*th one) is described through a continuous state variable $S_i(t) \in \mathbb{R}$, at each time step

 $t \in \mathbb{N}$. Each element can move on a two-dimensional lattice (Fig. 1A):

$$\Lambda(L) = \{(i,j) \mid 1 \le i, j \le L\}$$

$$\tag{1}$$

and the global state S_t of our automata set will be described by:

$$\mathbf{S}_{t} = \{S_{1}(t), ..., S_{N}(t)\}$$
(2)

Now a set of rules is defined:

(a) Neural network structure: the automata are considered as some kind of "neurons" (ants) and interactions are described as in neural networks [13] by means of a sigmoidal function $\Phi(x)$. If $S_j(t)$ is a given automaton (the spatial dependence is omitted for simplicity), the new states are updated following:

$$S_i(t+1) = \Phi\left[g\left(\sum_{j\in B(i)} J_{ij}S_j(t) - \Theta_i\right)\right]$$
(3)

where the sum is performed over the eight nearest automata (the boundary B(i)) but also the *i*th automaton itself:

$$S_{i}(t+1) = \Phi \left[g \left(J_{ii}S_{i}(t) + \sum_{i \neq j \in B(i)} J_{ij}S_{j}(t) - \Theta_{i} \right) \right]$$

$$(4)$$

with $J_{ii} \neq 0$. For simplicity we use the threshold $\Theta_i = 0$.

(b) Spontaneous activation: each automaton can be either *active* or *inactive* and, if active, it moves randomly to one of the eight nearest cells (if no space is available, no movement takes place). In our model a given automaton will be active if $S_i(t) > \theta$ and inactive otherwise. Here this threshold is $\theta = 10^{-16}$. Once a CA becomes inactive, it can return to the active state (with a *spontaneous activity level* S_a ; here we use $S_a = 0.1$) with some probability p_a .¹

(c) Coupling matrix: the coupling matrix J is not fixed. Connections are local and changing over time

as a consequence of movement. They are also statedependent i.e. J_{ij} will be a simple function of the states of the actually interacting pair (i, j) of automata, i.e.

$$J_{ij} = f(S_i^t, S_j^t) \tag{5}$$

In our case, where two basic states are defined, i.e. "active" and "inactive", the connection matrix reduces to the following 2×2 table:

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

At a given time step, the interaction J_{ij} between the *i*th and the *j*th elements is equal to $\lambda_{kl} \in \Lambda$ by depending of the activity states of the given elements. More precisely, J_{ij} will be equal to: λ_{11} when both ants are active, to λ_{12} , λ_{21} when one is active and the other inactive and to λ_{22} if both automata are inactive. In this paper we take for simplicity $\lambda_{ij} = 1$, but qualitatively similar results are obtained when $\lambda_{11} > 0$, λ_{12} , $\lambda_{21} > 0$ and $\lambda_{22} \ge 0$. Our choice is based in the observation of ant colonies. Self-interaction and positive feed-back (with local excitability) play an essential role in colony dynamics [15].

Here we take $\Phi(z) = \tanh(gz)$. When a given individual is in isolation, the previous equation (4) reads

$$S_i(t+1) = \tanh[\mu S_i(t)]$$
(6)

(here $\mu = gJ_{ii}$, and $J_{ii} > 0$) and it can easily be shown that for $\mu < \mu_c = 1$, a single point attractor is observed: $S_0 = 0$. For $\mu > \mu_c$, a saddle-node bifurcation occurs and two possible symmetric states S_{\pm} are available. In the linear approximation, i.e. S(t+1) = $[\partial_s \Phi(0)]S(t)$ for $S = S_0$, we obtain S(t+1) = $\mu S(t)$. If we define activity as previously, starting from $S(0) = S_a$ the state after τ iterations will be $S(\tau) = S_a \mu^{\tau}$, so the time for a given element to inactivate $(S(\tau) \leq \theta)$ will be

$$\tau = \frac{\log(\Gamma)}{\log(gJ_i)} \tag{7}$$

where $\Gamma = (\theta/S_a)^{-1}$. We can see that for a given Γ , as $\mu = gJ_i$ approaches μ_c the transient time τ increases, with a singularity at $\mu = \mu_c$. Let

$$N_{+} = \sharp \{S_i > \theta\} \tag{8a}$$

¹ A different possibility can be the removal and introduction of elements, as it happens for example in the immune system [14].



Fig. 2. Dynamical patterns corresponding to different phases; parameters as before but: (a) $\rho = 0.60$, g = 0.01; (b) $\rho = 0.25$, g = 0.03; (c) $\rho = 0.20$, g = 0.10; (d) $\rho = 0.80$, g = 0.10; (e) $\rho = 0.40$, g = 0.10; (f) $\rho = 0.50$, g = 0.40.

 $N_{-} = N - N_{+} = \sharp \{S_{i} \le \theta\}$ (8b)

If no spontaneous activation is introduced, in these conditions we will move towards a global attractor with no active elements, i.e. $\{S_i(\infty) = 0\}$; $\forall i = 1, ..., N$ when $\mu < \mu_c$ and to two alternative global states (via symmetry-breaking) otherwise when elements are mobile. For a frozen lattice, we expect to find all the elements belonging to each cluster in a given state and for $L \to \infty$ the order parameter $\Omega = N_+ - N_-$ goes to zero. At the other extreme, it was shown in a previous paper [18] that for $J_{ii} = 0$ (i.e. no self-interaction), the automata behaves randomly, with statistically equal numbers of active and inactive elements ($\Omega = 0$). Between both extremes, i.e. frozen clusters or random, featureless patterns, other macroscopic states are available when rule (b) is applied.

The additional rule of spontaneous activation makes possible to maintain our system in far from equilibrium conditions. In doing this, nonequilibrium constraints can drive our system near some critical state where complexity (and computation) is expected to emerge.

In previous papers [16–19] we have shown that several dynamical regimes can be observed. Here we study these regimes by means of two different parameters: the gain parameter (g) and the density of automata $\rho = N/L^2$, being the other fixed. Some general results can be mentioned and are roughly summarized in Fig. 2 for several parameter combinations (see Ref. [18] for a detailed discussion). For low densities, the total number of active elements over time, i.e. $N_+(t)$ changes chaotically. As ρ increases, these oscillations become more and more ordered, provided that g is not too large nor too low. The gain parameter plays an essential role in defining the period of oscillations. Some dynamical regimes have yet been studied in previous papers [16,18], with very good agreement with real data. Here we are interested in obtain a detailed parameter space for our model, as well as to give an interpretation of the observed oscillations in *Leptothorax* colonies. As will be shown below, the boundaries separating different dynamical phases have quite interesting critical properties.

3. Entropy, chaos and phase transitions

In order to characterize the macroscopic behaviour of our system, several measures from information theory can be used. It can be shown that such quantities can in fact serve as measures of complexity [1,5,20] and have been chosen for their ability to detect the onset of correlations near phase transitions. These measures will also help us in characterize the parameter space of the FNN. In this section the Shannon- Kolmogorov (SK) entropy [20] will be analysed. Here we are interested in the SK entropy as a measure of the diversity of macroscopic states, which can be measured from an adequate set of probabilities { $p(j, \rho)$ }.

Let T_j the number of time steps where exactly *j* elements where active (j = 0, 1, ..., N). Now, if *T* is the total number of time steps, the relative frequency of *j* simultaneously active elements will be $p(j, \rho) = T_j/T$. The SKE is then defined as:

$$S(\rho) = -\sum_{i=0}^{N} p(i,\rho) \log p(i,\rho)$$
(9)

 $S(\rho)$ is upper bounded by $S^{\max}(\rho) = \log(N)$ (if all states are equally available) and by $S^{\max}(\rho) = 0$ if only one state is present. The first case is linked with a completely random pattern. If a critical phenomenon takes place at intermediate densities, such situation will be related with a *complex* pattern, between both extremes. The Shannon entropy provide us with a first approach to the complexity of such dynamical patterns. In our case, it is expected that for low densities $S(\rho)$ will grow until the appearance of collective



Fig. 3. Shannon-Kolmogorov entropy $S(\rho)$ calculated from Eq. (9), for several (ρ, g) -values. Here L = 10 and $T = 10^4$ steps have been used after 10^3 transients were discarded.

phenomena. Beyond the onset of global oscillations, $S(\rho)$ will decrease by means of correlations.

In Fig. 3 we can see a characteristic maximum in the SK entropy for $g \in (0.05, 0.12)$ and $\rho \in (0.05, 0.65)$. Here $T = 10^4$ time steps were used after 10³ transients were discarded. Such a maximum takes place at a given $\rho_c \approx 0.18 - 0.20$ and separates, through a smooth transition, the domain of irregular fluctuations linked with low densities from that of regular oscillations. In order to make more obvious the existence of a phase transition, an order parameter can be defined. Here we take $\Gamma(\rho)$, defined as

$$\Gamma(\rho) = \log(N) - S(\rho) \tag{10}$$

which is zero if the elements are independent (when $S(\rho) \approx \log(N)$) but will be nonzero if $S(\rho)$ decreases as a consequence of self-organization. In Fig. 4a we see an example of the SK entropy for $\rho \in (0, 0.9)$ when g = 0.05. In Fig. 4b the corresponding order parameter $\Gamma(\rho)$ also shows a transition zone between two states.

Using the SK entropy, an extensive study of the (ρ, g) -space shows four dynamical domains, separating random behaviour, chaotic attractors, steady states



Fig. 4. (a) Entropy $S(\rho)$ versus density for g = 0.08; other parameters as before. (b) Phase transition shown by the order parameter $\Gamma(\rho)$, corresponding to data from (a).



Fig. 5. Phase space of the fluid neural network model. Four zones are clearly separated, as indicated. Parameters are the same as in Fig. 3 (see text).

and periodic oscillations (see Fig. 5). For high gvalues, the automata collapses to a steady state where all individuals are active and the FNN remains in a fixed point. If lower g are used, coherent oscillations are obtained, as mentioned before. And as g is more and more reduced, such coherent behaviour becomes more and more irregular. The boundary separating the chaotic domain from oscillations is determined through the maximum entropy. This (ρ, g) -points are those where the maximum of $S(\rho)$ is reached.



Fig. 6. Phase transition in the critical density as a function of g (see text). Beyond $g_c = 0.025$ the critical density where the maximum entropy is observed decreases with g, but for $g < g_c$ the maximum entropy is observed at $\rho = 1$.

For very small gain values $g < g_c = 0.025$ no phase transition arises. Here the entropy allways grows with ρ as the logarithm of the number of objects: $H \approx \log(N)$. All states tend to be equally represented i.e. $p(i, \rho) = 1/N$; the automata are roughly independent and in this sense the network is "random". Such a sharp transition is given in Fig. 6 where the ρ associated with the maximum entropy is plotted against g. For $g > g_c = 0.025$, the critical density decreases with g as expected. For $g < g_c$ the maximum entropy is reached for $\rho = 1$.

Now in order to characterize the existence of information transfer among elements, the joint information will be used in the next section.

4. Information at the edge of chaos

The maximum entropy obtained in the previous section is linked, as shown by Haken [23] to critical fluctuations. At the critical density the global activity starts to control the individual automata, and the fluid neural network begins to show collective behaviour. Now we will extend our discussion to emergent computational properties, and other macroscopic quantities from information theory must be used.

In order to determine if two elements cooperate in the support of computation, the mutual information [1,20] between two mobile automata will give us a characterization of such information exchange. For two arbitrary elements, say the *i*th and the *j*th ones, we can follow the dynamical evolution of their states, i.e.

$$O_{\beta}^{i} = \left\{ S_{i}(t), ..., S_{i}(t+m) \right\}$$
(11)

$$O_{\beta}^{j} = \left\{ S_{j}(t), ..., S_{j}(t+m) \right\}$$
(12)

during $m = 10^4$ time steps (after 10^3 transients are discarded). Here β indicates a set of parameters (i.e. $\beta \equiv (g, p_a, S_a, \theta)$). We will restrict our attention to the new sets $A^i_\beta(\rho)$, $A^j_\beta(\rho)$ whose elements $a_i(t)$ are defined as $a_i(t) = 1$ if $S_i(t) > \theta$ and $a_i(t) = 0$ otherwise, and are in fact the observed states in experimental situations (say active/inactive ants). Thus we define

$$A_{\beta}^{i} = \left\{ a_{i}(t), ..., a_{i}(t+m) \right\}$$
(13)

$$A_{\beta}^{j} = \left\{ a_{j}(t), ..., a_{j}(t+m) \right\}$$
(14)

which provides a description in terms of an "alphabet" involving only two letters (we have a markovian coarse-graining [5]).

Let $P_i(r)$ the probability of finding the *i*th automata in the *r*th state $(r \in \{0, 1\})$ and let $P_{ij}(r, s)$ be the joint probability of find simultaneously the *i*th automata in the state *r* and the *j*th one in the state *s*, i.e. $P_{ij}(r, s) = P[\{a_i = r\} \cap \{a_j = s\}]$. All probabilities were calculated using *m* time steps. The information transfer between these elements will be

$$I_{\beta}(A_i, A_j) = H_{\beta}(A_i) + H_{\beta}(A_j) - H_{\beta}(A_i, A_j) \quad (15)$$

being $H_{\beta}(A_i)$ the Shannon entropy of the *i*th object, i.e.

$$H_{\beta}(A_i) = -\sum_r P_i(r) \log_2 P_i(r)$$
 (16)

$$H_{\beta}(A_j) = -\sum_{r} P_j(r) \log_2 P_j(r)$$
 (17)

and $H_{\beta}(A_i, A_j)$ the joint entropy, defined as:

$$H_{\beta}(A_{i}, A_{j}) = -\sum_{r} \sum_{s} P_{ij}(r, s) \log_{2} P_{ij}(r, s)$$
(18)

Some relevant inequlatities can be easily obtained [20] in particular

$$0 \le I_{\beta}(A_i, A_j) \le H_{\beta}(A_i) + H_{\beta}(A_j)$$
(19)

since $H_{\beta}(A_i, A_j) \geq 0$. Following two given individuals over time, we calculate the entropy and information transfer between both automata. In Figs. 7a,b some of our results are sumarized. Here for small densities $\rho \ll \rho_c$, we have $P_i(1) \ll P_i(0)$; as a consequence of diluted interactions and the small probability of activation, $H_{\beta}(A_i)$ is small. When ρ is large $(\rho \to 1)$ we have $P_i(0) \ll P_i(1)$ because of global activation. At ρ_c , we have $P_i(0) \approx P_i(1)$ and the individual entropy reaches a maximum. The joint entropy $H_{\beta}(A_i, A_j)$ always decreases with ρ : the joint probability $P_{ij}(1, 1) \to 1$ as $\rho \to 1$.

As a consequence, a maximum in information transfer is obtained, separating the ordered and disordered domains, as expected when computation emerges at the edge of chaos. For a given set of parameters the $I_{\beta}(A_i, A_i)$ plot is given. At $\rho = \rho_c$ (here $\rho_c \approx 0.18$) the onset of ordered dynamics matches the maximum information transfer. Following previous studies, we also plot the entropy-information transfer picture. As shown in Fig. 7b, we observe a maximum value of information (the one observed at the critical point) for a given entropy, say H_{β}^{c} . For high entropies, i.e. $H_{\beta} >$ H_{B}^{c} , information transfer through the system is difficult because low densities make contact sparse and a given activation event do not spread. For low density values, where $H_{\beta} < H_{\beta}^{c}$, coherent behavior gives birth to memory effects: after a given activation, information spreads but individuals remain at the same state for a long time. At the edge between both tendencies, the entropy is sufficiently low to store information but sufficiently high to enable information to propagate. The recent evidence from ant colonies where the critical density matches the values obtained in our automata can be the fingerprint of emergent computation at the edge of chaos [21].



Fig. 7. (A) Information transfer $I_{\beta}(A_i, A_j)$ (parameters as indicated): it is obtained from Eq. (15), following two randomly chosen automata over 10⁶ time steps. A maximum is reached at the critical density $\rho_c \approx 0.18$. (B) Information transfer $I_{\beta}(A_i, A_j)$ versus entropy $H_{\beta}(A_i)$ (16); a maximum in I_{β} is obtained for $H_{\beta} \approx 0.42$. The maximum separates, as before the regions of low-dimensional chaos from the periodic attractors.

5. Transient length

An additional dynamical measure of the edge of chaos is provided by means of the transient length. Transients diverge in the vicinity of critical points [22]. If a probabilistic description is used, a steady probability distribution can be calculated (say $\{P_s(r)\}$) by averaging over a long number of time steps. We can also calculate a time-dependent set of probabilities $\{P(r;t)\}$ by following our system over time. In such a way the second set tends asymptotically to $\{P_s(r)\}$ and we can measure how many time steps are necessary in order to have enough information (less uncertainty) about our system.

In our study we will use the so called information gain (or Kullback information) [5,23] defined as

$$K_{\beta}(t) = \sum_{r} P(r;t) \log \left[\frac{P(r;t)}{P_{s}(r)} \right]$$
(20)

being $\{P_s(r)\}$ the stationary probability distribution (calculated from $m = 10^4$ steps) and $\{P(r; t)\}$ the probability distribution calculated until the *t*th time step. As previously defined, $r \in \{0, 1\}$. Obviously, we have

$$\sum_{r} P(r;t) = 1, \quad \sum_{r} P_s(r) = 1$$

and $K_{\beta}(t)$ has also the important property $K_{\beta}(t) \ge 0$ where equality holds if and only if $P_s(r) = P(r; t)$ for all r. Information gain is a very useful measure in systems far from equilibrium described by probability measures. It has been used for example as a quantitative characterization of phase transitions in lasers as well as in neural computation [23].

The time variation of $K_{\beta}(t)$ can be obtained by deriving the previous definition (20) and we obtain the equation:

$$\frac{\partial K_{\beta}}{\partial t} = \sum_{r \neq r'} \left[\omega(r|r') P(r';t) - \omega(r'|r) P(r;t) \right] \log \left[\frac{P(r;t)}{P_s(r)} \right]$$
(21)

where $\omega(r|r')$ are the transition probabilities of the master equation

$$\frac{\partial P(r,t)}{\partial t} = \sum_{r'} \left[\omega(r|r')P(r';t) - \omega(r'|r)P(r;t) \right]$$
(22)

In our model, P(r, t) is the probability of find the automaton in active (r = 0) or inactive (r = 1) state, after t time steps. Here $\{\omega(r|r')\}$ is the set of transition probabilities between these states.



Fig. 8. Transient length τ obtained from (23) using n = 100 samples. The information gain $K_{\beta}(t)$, defined in (20) was estimated following a single automaton over τ time steps. This transient length becomes maximum at the critical density.

It can be shown from the positivity of these probabilities [5] that

$$\frac{\partial K_{\beta}}{\partial t} \le 0 \tag{23}$$

i.e. information gain always decreases with time. This inequality holds equally well in isolated and nonisolated systems subjected to permanent nonequilibrium constraints [5]. At this probabilistic level, the problem of multiplicity of solutions and instability simply does not arise.

Starting from an arbitrary initial condition, we first calculate the stationary distribution for the *i*th automaton i.e. we calculate $\{P_s(0), P_s(1)\}$ over the coarse grained trajectory A_β , as previously defined. Using *m* time steps, such stationary distribution is reached and the time-dependent probabilities $\{P(0; t), P(1; t)\}$ are then estimated. The transient length τ is defined as the first time step such that

$$K_{\beta}(\tau) \le \epsilon \tag{24}$$

being here $\epsilon = 0.0025$. As shown in Fig. 8, τ shows also a maximum value at $\rho \approx 0.2$ for the parameters given. Below the critical point, the automata remain inactive for long periods of time and they change state essentially by means of random activation. Beyond the critical point, the automata remain now active for a long time because frequent interactions. In both cases, one of the extreme stationary probabilities wins over the other one, and it becomes more easy to reach the steady distribution. For the critical point, a mixture between both dynamical regimes appears and as a consequence the probability distribution spreads out (we have greater diversity of activity patterns).

This result is an additional support of our conjecture. At the critical point, it takes longer to describe what is going on than it does when far from ρ_c . As in elementary CA [1,2], the critical boundary supports both static (uniform patterns) and propagating structures (waves of activation). Both patterns form the basis for signals and storage.

6. Discussion

In this paper, several properties of FNN have been studied. It is our belief that this automaton can be a very useful framework for several complex systems, such as insect societies or the immune system. As in standard NN, where the elements are oversimplified structures, nevertheless network computation has proved to be robust to noise and changes in detailed rules. Concerning insect societies, Wilson stated: "an individual social insect processes less information than an individual solitary insect, but as part of an aggregative activity, the social insect contributes to more complex computation. The colony works as a single organism" [24]. In other words, very simple automata can be used as the basis for our network. Using phase transitions, a given set of these automata become a "single organism".

At the critical density ρ_c (for $g > g_c \approx 0.025$) the entropy $S(\rho)$, as well as the information transfer I_β become maximum. In this parameter space, adding elements to our FNN shows a clear deviation from the maximum entropy ($H \approx \log(N)$, as expected for independent automata) and close to the instability point information is able to increase due to critical fluctuations. In a recent study [25] it has been shown that $f^{-\beta}$ fluctuations are in fact present in this critical domain, as expected for a system in a critical state, as the dynamical counterpart of our results. Such fluctuations are also observed in ant colonies [25].

The maximum information transfer I_{β} also shows the point where computational properties can emerge, at the edge between the ordered and the chaotic phases. Interestingly, the key parameter is nothing but the number of automata involved. In *Leptothorax* ant colonies, it has been shown that certainly the number of individuals is not arbitrary: as the critical density $\rho_c \approx 0.2$ is approached, ants change the colony boundaries to reach again the critical value [21]. Other investigations have also shown, now in other species of ants, that contact rate regulates the ant network [26], playing a crucial role in macroscopic patterns of behavior, as well as in the spread of information [27].

As early shown by Haken [23] fluctuations drive nonlinear systems to explore new states. In our system such fluctuations are present in terms of random activation events and random movement in space. In [23] it was also shown that the performance of a selforganizing system will depend on a compromise between: (a) large fluctuations on flat potential curves (adaptability) and (b) small fluctuations in deep potential valleys (in order to have reliability). In Haken's view, it was conjectured that the size of fluctuations is controled by means of the number of components. In *Leptothorax* ant colonies, such a control of reliability and adaptability by means of density is observed, and a theoretical interpretation is provided through our model.

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