WHAT IS RHYTHMIC APPLAUSE ?

- a positive manifestation of the spectators after an exceptional performance
- spectators begin to clap in phase (synchronization of the clapping)
- appears after the initial thunderous unsynchronized clapping
- disappear and reappear randomly several times during the applause
- a phenomenon resembling phase-transitions

thunderous		synchronized
uncorrelated	$\longleftarrow \longrightarrow$	correlated
applause		rhythmic applause

- characteristic for smaller and culturally more homogeneous eastern European communities
- it happens sporadically in Western and American audiences
- it is considered to be the human-scale example of the synchronization processes known in numerous natural systems
- how I got interested ? - I took seriously my friends (Y. Brechet) joke
- what we are interested in: CAN THE PHENOMENON BE UNDERSTOOD AND DESCRIBED BY THE METHODS OF STATISTICAL PHYSICS ????
- the answer is YES: this is the talk about.......

OTHER SYNCHRONIZATION EXAMPLES IN NATURE

- 1667 Christiaan Huygens: synchronization of pendulum clocks hanging on a wall
- networks of coupled Josephson junctions
- synchronized flashing of fireflies (along the river-sides in Thailand)
- synchronization of the chirping of crickets
- neural cells of the brain synchronize voltage fluctuations
- pacemaker cells in the heart synchronize their fire
- womens living together find their menstrual cycle synchronized

WHY DOES SYNCHRONIZATION APPEAR?

- synchronization of identical oscillators coupled by phase-minimizing interactions is obvious
- HOWEVER!!! biological (and even real physical) objects ARE NOT IDENTICAL!
- QUESTION: Can a group of globally coupled non-identical oscillators synchronize ? - if yes, under what conditions ?
- STUDIES: Winfree (1967), Kuramoto and Nishikva (1987), Strogartz and Mirollo (1990)

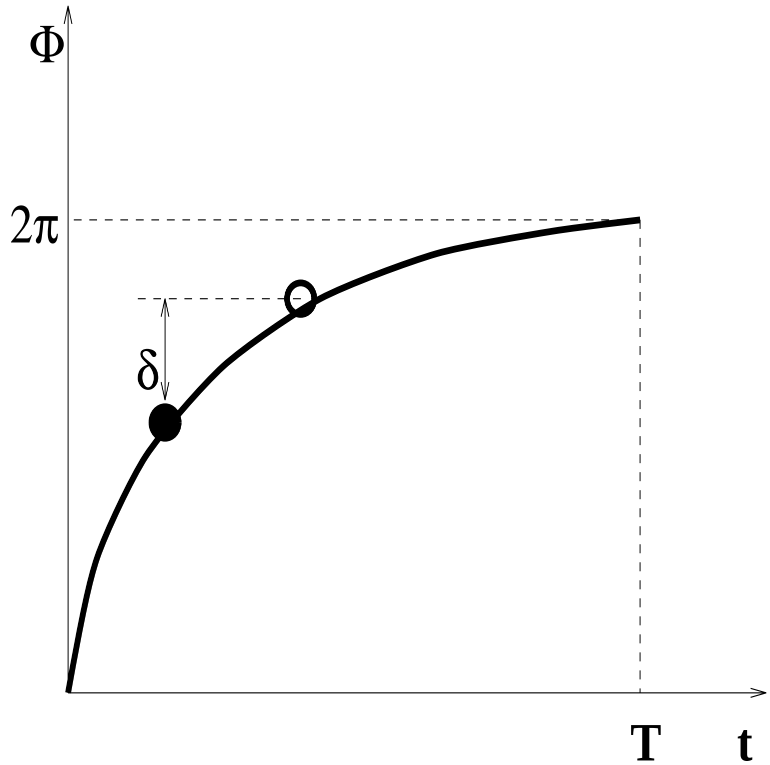
MODELS FOR SYNCHRONIZATION PROBLEMS

1. mathematically coupled maps (usually small number of identical oscillators) ref.: *W. Just, Phys. Rep.* **290**, *101 (1997)* - intersting mainly from the view-point of dynamical systems
2. pulse-coupled oscillators - **integrate and fire** (I-F) type models
3. phase-coupled rotators - **the Kuramoto model**

INTEGRATE ANF FIRE OSCILLATORS WITH GLOBAL COUPLING

- introduced by S. Strogatz and R. Mirollo, *SIAM J. Appl. Math.* **50**, *1645 (1987)*, review by S. Bottani *Phys. Rev. E* **54** *2334 (1997)*
- main features:
  - we have N oscillators whoose Φ_i phase evolve nonlinearly as a function of time
  - the time evolves discretely by dt time-steps.
  - when $\Phi_i \geq 2\pi \rightarrow \Phi_i = 0$ and stays there untill the next time step. If $\Phi_j \neq 0$ ($j = \overline{1, N}$, $j \neq i$) $\rightarrow \Phi_j = \Phi_j + \delta$ ($\delta \geq 0$). (i.e the effect of one fireing is to increase the phase of all oscillators with $\Phi \neq 0$ by δ).
  - a time step is ended when the phase of no more oscillator evolves
  - oscillators are considered synchronized if they fire in the same time-step (in the same avalanche)
  - interactions are only during the firing process, so pulse-like

a general $\Phi(t)$ evolution curve:



Results for the model:

- analytical results available for the case of oscillators with identical natural frequencies and random initial phases
  - for concave down and linear $\Psi(t)$ curves always synchronize
  - for concave up $\Psi(t)$ curves no synchronization
- for non-identical oscillators some approximations and computer simulations:
  - synchronization is possible only for linear and concave down $\Psi(t)$ curves
  - synchronization is continiously destroyed by increasing the dispersion

2. THE KURAMOTO MODEL

- a system of globally coupled rotators, with a $g(\omega)$ distribution of their natural frequencies.
- the interaction is of the same strength with each rotator, and favorize phase synchronization:

$$W_k = \frac{K}{N} \sum_{j=1}^{N} \sin(\phi_j - \phi_k) \tag{1}$$

- the equation of motion: N coupled non-linear differential equations:

$$\frac{d\phi_k}{dt} = \omega_k + \frac{K}{N} \sum_{j=1}^{N} \sin(\phi_j - \phi_k) \tag{2}$$

- the order parameter:

$$r(t) = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\phi_j(t)} \right| \tag{3}$$

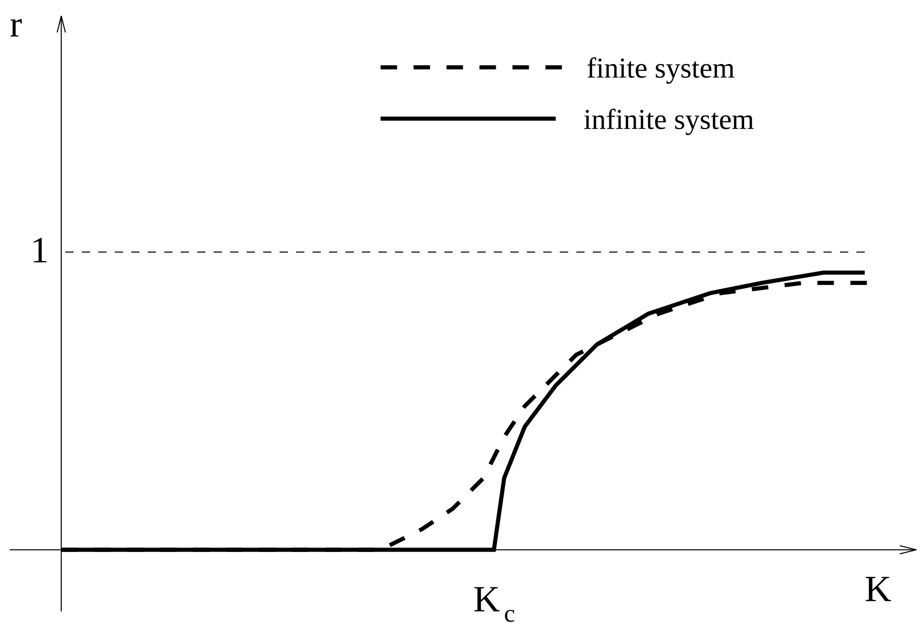
- for the chosen harmonic interaction the equations can be decoupled and thus the problem is EXACTLY solvable
- both equilibrium and transient phenomenon are exactly investigated: *Y. Kuramoto and I. Nishikawa; J. Stat. Phys.* **49**, *569 (1987)*

Main results of the Kuramoto model
(equilibrium dynamics)

- for $N \to \infty$ there is a **critical coupling**
  - for $K < K_c \to r = 0$, for $K > K_c \to r > 0$
  - a second-order phase transition-like phenomenon
- for a $g(\omega)$ Gauss-like distribution with D dispersion:

$$K_c = \sqrt{\frac{2}{\pi^3}}\, D \tag{4}$$

- the synchronization can be enhanced both by increasing K or decreasing D



I-F OR KURAMOTO MODEL VERSUS RHYTHMIC APPLAUSE

the I-F model

advantage:

- pulse-like interaction (resembles the sound of clapping)
- avalanches: resembles the coherent clapping
- sequences of differently synchronized regimes (usually periodical sequences)

problems:

- no memory effects incorporated (memory effects should be important in human clapping rhythm)
- synchronization should evolve from the beginning (no initial waiting time)

the Kuramoto model

advantage:

- memory effects can be approximated through the phase-coupling
- partially synchronized states ($r > 0$) approximates the rhythmic applause
- depending on the ratio K/D possible both synchronized and unsynchronized states

problems:

- phase coupling is not realistic
- for synchronized states r should increase from the beginning
- synchronization once achieved should not be lost

FIRST CONCLUSION: rhythmic applause is not described within a SIMPLE application of the I-F or Kuramoto model

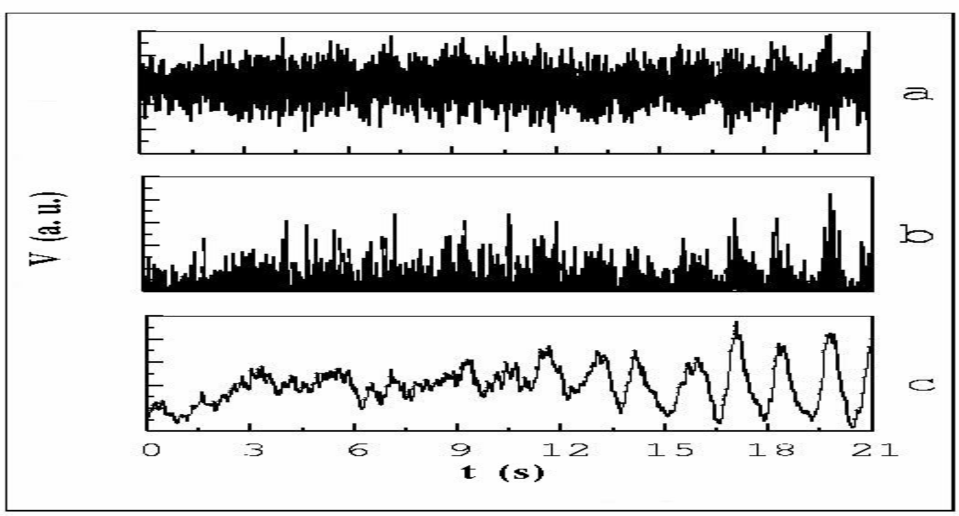
EXPERIMENTAL STUDY

- a. recording global and local sound during the applause, analyzing it in different ways
- b. well-controlled clapping experiments on a group of students and a systematic study on one person

a. Recordings, analysis, results

digitization method

- a. recorded signal,
- b. square relative to the average level of a.
- c. short-time (0.2 s) average of b.



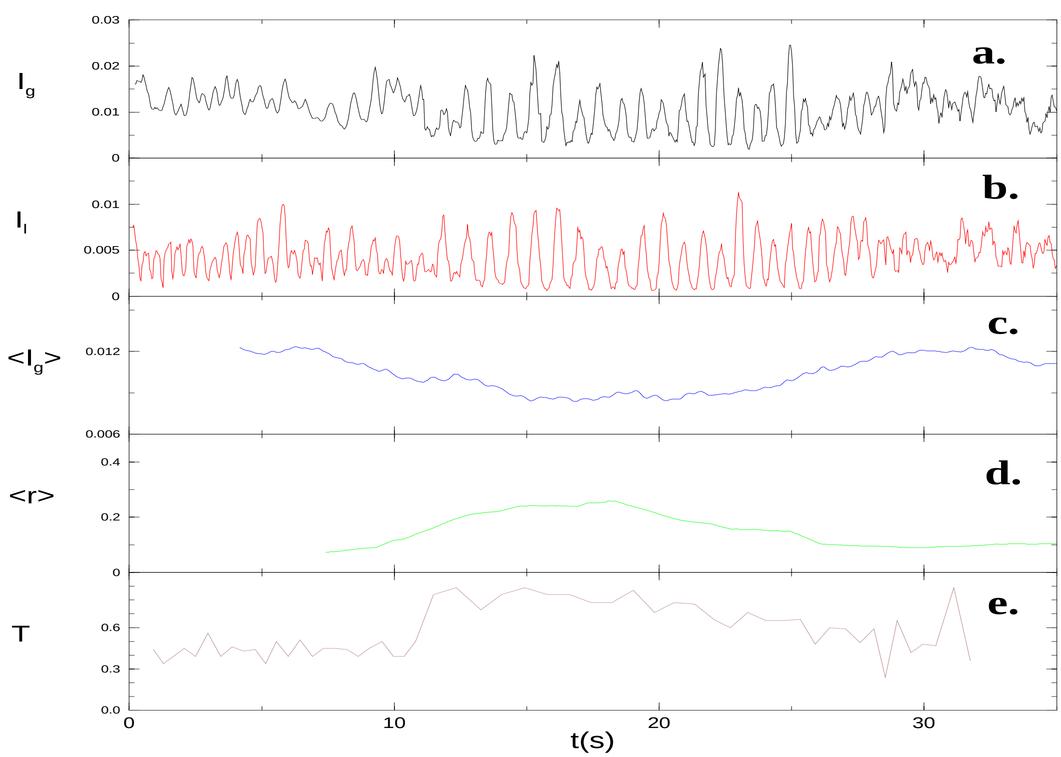
characteristic result:

- a. global signal
- b. local signal
- c. long-time (3s) averaged signal
- d. experimental order parameter:

$$r_{exp}(t) = \max_{\{T,\Phi\}} \frac{\int_{t-T}^{t+T} s(T) \sin(2\pi/T + \Phi) dt}{\int_{t-T}^{t+T} s(t) dt} \qquad (5)$$

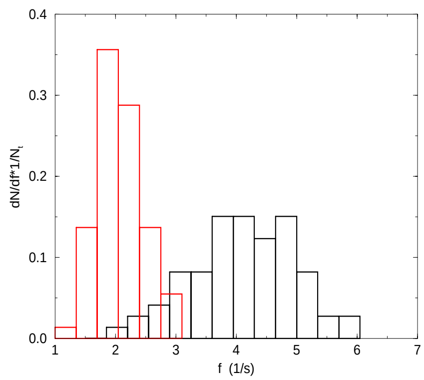
($\Phi \in [0, 2\pi]$, $T \in [0.1, 5]$s),

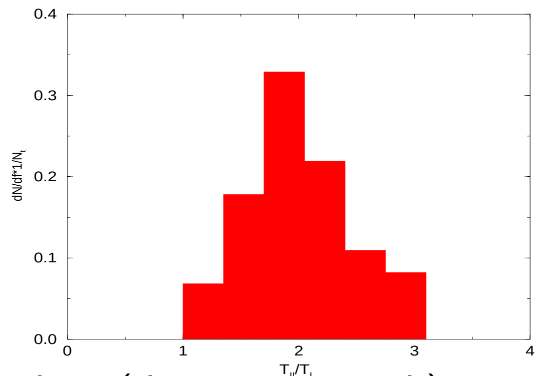
- e. time period between clapping for the chosen local applause



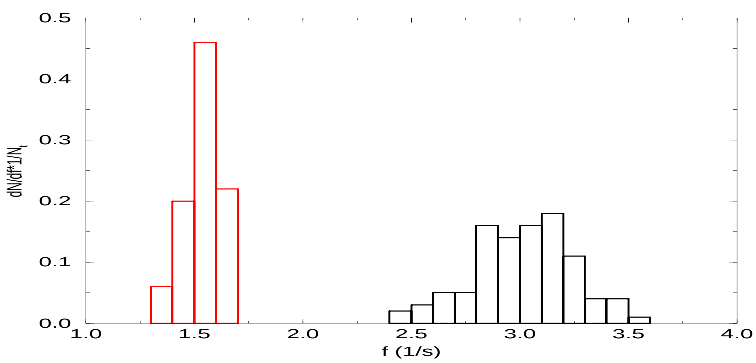
b. Controlled clapping experiments

- separated by the group, 73 high-school students were asked to clap in two modes:
  - mode I: the initial thunderous applause (distribution of the frequencies plotted with black)
  - mode II: during rhythmic applause (distribution plotted with red)





- 100 experiments on one student (during one week)



- **results:** two clearly distinguishable clapping modes, Gauss-type distribution of the frequencies for each; the ratio of mean medium frequencies $\simeq$ 2 and the ratio of dispersions $\simeq 1/2$; the ratio of the frequencies for the two clapping modes a Gaussian distribution centered around 2; clapping modes quite-stable

during rhythmic applause the spectators double their clapping period !!!!

WHAT HAVE WE LEARNED FROM EXPERIMENTS

- there are two distinct clapping modes:
  - high frequency clapping (mode I), before and after rhythmic applause (large dispersion of the clapping frequencies)
  - low frequency clapping (mode II) during rhythmic applause (dispersion of the clapping frequencies reduced to half)
- during rhythmic applause the long-time averaged noise intensity decreases, while the order parameter increases
- during unsynchronized clapping the order parameter is small (no synchronization), but the average noise intensity is big.
- the clapping frequency of the individual is increased before synchronization is lost

RESULTS IN THE VIEW OF KURAMOTO MODEL

- all results are understandable by simply applying the Kuramoto-Nishikava result:

$$K_c = \sqrt{\frac{2}{\pi^3}}\, D \qquad (6)$$

  (similar expressions are valid for non-harmonic coupling, $K_c \sim D$)
- K is imposed by social and human parameters (fixed), synchronization is achieved by reducing D (shifting to mode II clapping)
- why synchronization is lost ??? – FRUSTRATION IN THE SYSTEM (this is leading to the interplay of synchronized and unsynchronized regimes)

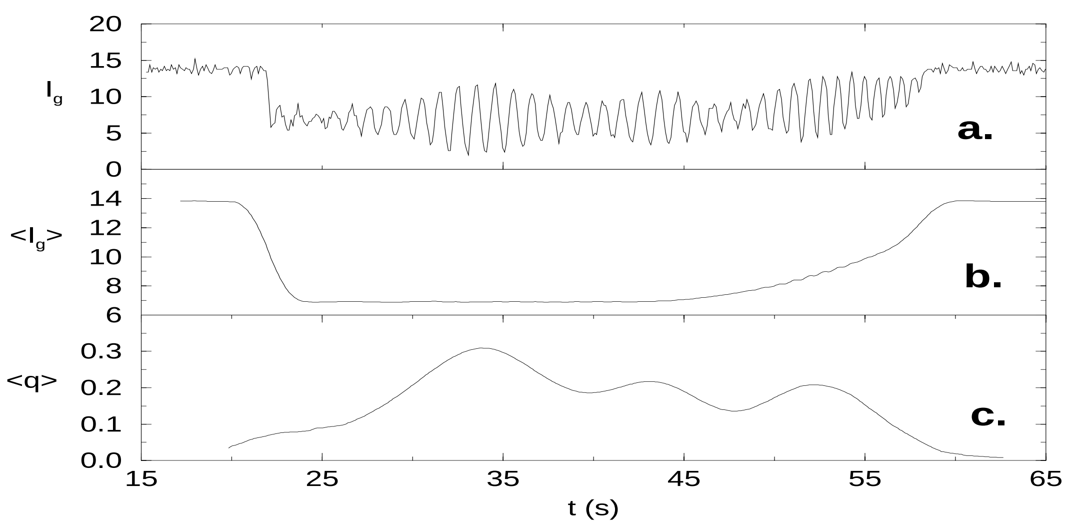
FRUSTRATION IN THE CLAPPING AUDIENCE

- what spectators want ????
  - big noise intensity (enthusiastic manifestation)
  - synchronization (clapping together)
- THE TWO DESIRES ARE CONFLICTING !!!
  - big noise intensity is achieved only by clapping faster i.e. mode I clapping, here no synchronization is possible (hitting stronger does not increase substantially the average noise level)
  - synchronization is achieved in mode II clapping (small dispersion of clapping frequencies), low noise intensity (fewer claps per unit time)
- the two main desires cannot be achieved in the same clapping mode $\rightarrow$ characteristic interplay of synchronized and unsynchronized regimes
- clapping audience after a good performance is frustrated in this sense

A COMPUTER EXERCISE ON THE KURAMOTO MODEL

- a simple computer simulation exercise on the Kuramoto model can give confidence in our results
- we considered:
  - $N = 70$, $K = 0.8s^{-1}$, $D = 2\pi/6.9s^{-1}$, $\overline{\omega} = 2\pi s^{-1}$
  - we associate a $\tau = 0.01s$ time-length and $\omega/\overline{\omega}$ intensity pulse for each oscillator passing through a multiple of 2π
  - at $t_1 = 21s$ we double the oscillators natural period
  - beginning with $t_2 = 35s$ we linearly increase the frequencies back to their original value
- the total noise intensity is plotted and analyzed

Result:



SOME COMMENTS

- the game of rhythmic applause "has to be learned" (it is known in all Eastern European countries)
- applauding the "great leader" speech in communist time (never destroyed synchronization). The audience was not frustrated, no real enthusiasm......
- never occurs in big open air concerts (very small coupling, period doubling is not enough)
- a good sign for the performance is not a continuous synchronized clapping, but an interplay between synchronized and unsynchronized regimes

CONCLUSION

- the scenario of rhythmic applause can be understood within a simple application of the Kuramoto model
- synchronization is achieved by period doubling the clapping rhythm
- the characteristic interplay between synchronized and unsynchronized regimes is due to a frustration in the system